The emergent scale symmetry and the sound velocity in the nuclear matter

arXiv:1604.06889 & arXiv:1611.09975

Won-Gi Paeng, RISP in IBS

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The emergent scale symmetry

In Paeng, Rho, arXiv:1611.09975(arXiv:1604.06889)
Lee, Paeng, Rho, PRD 92, no. 12, 125033 (2015)
Paeng, Lee, Rho, Sasaki, PRD 88, 105019 (2013), PRD 85, 054022 (2012)
Sasaki, Lee, Paeng, Rho, PRD 84, 034011 (2011)

- It was suggested that the infrared fixed point relevant to the emergent scale symmetry could be reached in dense nuclear matter. That is

$$\partial^{\mu} D_{\mu} = \theta^{\mu}{}_{\mu} = 0$$

in dense nuclaer matter under the scale transformation $x^{\mu} \rightarrow \lambda^{-1} x^{\mu}$ although the scale symmetry of QCD is broken in vacuum,

$$\left(\theta^{\mu}{}_{\mu}\right)_{QCD} = \frac{\beta(\alpha_s)}{4\alpha_s} G^{a}{}_{\mu\nu} G^{a\mu\nu} + \sum_{q} m_q \bar{q} q \neq 0.$$

- If the scale symmetry is restored in dense nuclear matter, we could construct the scale-invariant hidden local symmetric theory(sHLS) having σ as the Nambu-Goldstone boson of the spontaneous scale symmetry breaking joining into O(4) multiplet on the infrared fixed point with π as the pseudo Nambu-Goldstone boson of the spontaneous chiral symmetry breaking:

$$
\begin{aligned}
\mathcal{L}_{\rm inv} &= \mathcal{L}_N + \mathcal{L}_M \\
\mathcal{L}_N &= \bar{N} i \left(\partial_\mu - i g_\rho \vec{\rho}_\mu \cdot \frac{\vec{\tau}}{2} - i g_\omega \frac{\omega_\mu}{2} \right) N - \frac{m_N}{f_\chi} \chi \bar{N} N + g_A \bar{N} \gamma^\mu \alpha_{\perp\mu} \gamma_5 N \\
&\quad + g_{V\rho} \bar{N} \gamma^\mu \left(\alpha_{\parallel\mu} - g_\rho \vec{\rho}_\mu \cdot \frac{\vec{\tau}}{2} \right) N + g_{V\omega} \bar{N} \gamma^\mu \left(\frac{\partial_\mu \sigma_\omega}{2 f_{\sigma\omega}} - g_\omega \frac{\omega_\mu}{2} \right) N \,, \\
\mathcal{L}_M &= \frac{f_\pi^2}{f_\chi^2} \chi^2 {\rm tr} \left[\alpha_{\perp\mu} \alpha_\perp^\mu \right] + \frac{f_{\sigma\rho}^2}{f_\chi^2} \chi^2 {\rm tr} \left[\left(\alpha_{\parallel\mu} - g_\rho \vec{\rho}_\mu \cdot \frac{\vec{\tau}}{2} \right)^2 \right] + \frac{f_{\sigma\omega}^2}{2 f_\chi^2} \chi^2 \left(\frac{\partial_\mu \sigma_\omega}{f_{\sigma\omega}} - g_\omega \omega_\mu \right)^2 \\
&\quad - \frac{1}{2} {\rm tr} \left[\rho_{\mu\nu} \rho^{\mu\nu} \right] - \frac{1}{2} {\rm tr} \left[\omega_{\mu\nu} \omega^{\mu\nu} \right] + \frac{1}{2} \partial_\mu \chi \cdot \partial^\mu \chi
\end{aligned}
$$

at the lowest order of the hidden local symmetry, where

$$
\rho^{\mu\nu} = \partial^\mu \vec{\rho}^\nu \cdot \tfrac{\vec{\tau}}{2} - \partial^\nu \vec{\rho}^\mu \cdot \tfrac{\vec{\tau}}{2} - i g_\rho \left[\vec{\rho}^\mu \cdot \tfrac{\vec{\tau}}{2}, \vec{\rho}^\nu \cdot \tfrac{\vec{\tau}}{2} \right] ,
$$

$$
\omega^{\mu\nu} = \partial^\mu \tfrac{\omega^\nu}{2} - \partial^\nu \tfrac{\omega^\mu}{2}
$$

ρ and ω are included as the gauge bosons of the hidden local symmetry.

Here, we would like to include the terms to account for the trace anomaly of QCD,

$$\theta^{\mu}_{\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} q$$

with the gluon G and the quark q, which breaks the scale invariance. In terms of $U = \xi_L^{\dagger}\xi_R$ and χ, the scale symmetry breaking can be considered by the trace of the energy momentum tensor θ^{μ}_{μ} as

$$\theta^{\mu}_{\mu} = 4V(\chi) - \chi \frac{\partial V(\chi)}{\partial \chi} + \frac{f_\pi^2}{4} \mathrm{Tr}\left(MU^{\dagger} + h.c.\right) \left(\frac{\chi}{f_\sigma}\right)^3$$

which are given by

$$\mathcal{L}_{\mathrm{breaking}} = -V(\chi) + \frac{f_\pi^2}{4} \mathrm{Tr}\left(MU^{\dagger} + h.c.\right) \left(\frac{\chi}{f_\sigma}\right)^3$$

with the mass matrix M as diag$M = (m_\pi^2, m_\pi^2)$ for $SU(2)$.

The mean field approach to study the nuclear matter

$$\left.\frac{\Omega(T=0)}{V}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle} = \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4}\ln\left(\frac{E_F+k_F}{m_N^*}\right)\right] + V(\langle\chi\rangle)$$
$$+\left[g_\omega\left(g_{V\omega}-1\right)\langle\omega_0\rangle-\mu\right]\frac{2}{3\pi^2}k_F^3 - \frac{1}{2}f_{\sigma\omega}^2 g_\omega^2\frac{\langle\chi\rangle^2}{f_\sigma^2}\langle\omega_0\rangle^2$$

at zero temperature($T=0$), where $\langle\omega_0\rangle$ and $\langle\chi\rangle$ are the vev of $\omega_{\mu=0}$ and χ, $m_N^* = \frac{m_N}{f_\sigma}\langle\chi\rangle$ and $E_F = \sqrt{k_F^2 + m_N^{*2}}$. When we consider the thermodynamic relation,

$$\Omega = E - TS - \mu N$$

with the entropy S and the particle number N, the nucleon number density is given by

$$n \equiv N/V = -\frac{\partial(\Omega/V)}{\partial\mu} = \frac{2}{3\pi^2}k_F^3$$

and the chemical potential μ is given by requiring the condition

$$\frac{\partial(\Omega/V)}{\partial n} = 0$$

so that

$$\mu = E_F + g_\omega\left(g_{V\omega}-1\right)\langle\omega_0\rangle\,.$$

Then, the energy density ϵ and the pressure P at $T = 0$ are given by

$$\begin{aligned}\epsilon &= \frac{1}{4\pi^2}\left[2E_F^3 k_F - m_N^{*2} E_F k_F - m_N^{*4} \ln\left(\frac{E_F + k_F}{m_N^*}\right)\right] \\ &\quad + g_\omega\left(g_{V\omega} - 1\right)\langle\omega_0\rangle n - \frac{1}{2} f_{\sigma\omega}^2 g_\omega^2 \frac{\langle\chi\rangle^2}{f_\sigma^2}\langle\omega_0\rangle^2 + V(\langle\chi\rangle)\end{aligned}$$

and for a homogeneous matter,

$$\begin{aligned}P &= -\left.\frac{\Omega}{V}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle} \\ &= \frac{1}{4\pi^2}\left[\frac{2}{3}E_F k_F^3 - m_N^{*2} E_F k_F + m_N^{*4} \ln\left(\frac{E_F + k_F}{m_N^*}\right)\right] \\ &\quad + \frac{1}{2} f_{\sigma\omega}^2 g_\omega^2 \frac{\langle\chi\rangle^2}{f_\sigma^2}\langle\omega_0\rangle^2 - V(\langle\chi\rangle)\,.\end{aligned}$$

$$\left.\frac{\partial \Omega}{\partial \chi}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle} = 0\,, \qquad \left.\frac{\partial \Omega}{\partial \omega_0}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle} = 0\,.$$

They are written as

$$\frac{m_N^2\langle\chi\rangle}{\pi^2 f_\sigma^2}\left[k_F E_F - m_N^{*2}\ln\left(\frac{k_F+E_F}{m_N^*}\right)\right] - \frac{f_{\sigma\omega}^2}{f_\sigma^2} g_\omega^2 \langle\omega_0\rangle^2\langle\chi\rangle + \left.\frac{\partial V(\chi)}{\partial \chi}\right|_{\chi=\langle\chi\rangle} = 0\,,$$

$$g_\omega\left(g_{V\omega}-1\right)n - f_{\sigma\omega}^2 g_\omega^2 \frac{\langle\chi\rangle^2}{f_\sigma^2}\langle\omega_0\rangle = 0\,.$$

Interestingly, if we calculate

$$\begin{aligned}\langle\theta_\mu^\mu\rangle &= \langle\theta^{00}\rangle - \sum_i \langle\theta^{ii}\rangle \\ &= \epsilon - 3P\end{aligned}$$

, we find that

$$\langle\theta_\mu^\mu\rangle = 4V(\langle\chi\rangle) - \langle\chi\rangle \left.\frac{\partial V(\chi)}{\partial\chi}\right|_{\chi=\langle\chi\rangle}$$ in nuclear matter

$$\theta_\mu^\mu = 4V(\chi) - \chi\frac{\partial V(\chi)}{\partial \chi}$$ in vacuum

Dilaton Limit Fixed Point in nuclear matter

It was shown in terms of Wilsonian renormalization group (RG) equations that there is a possible infrared fixed point in sHLS with baryons included (bsHLS for short)

$$\boxed{(m_N,\ g_{V\rho} - 1,\ g_A - g_{V\rho}) \to (0,\ 0,\ 0),}$$

in vacuum

which is called Dilaton-Limit Fixed Point(DLFP) for the nucleon mass m_N, the ρNN coupling $g_{\rho NN} = (g_{V\rho} - 1)\, g_\rho$–with g_ρ the hidden gauge coupling–and g_A for the axial-vector current.

After taking $\sigma \to \langle\sigma\rangle + \sigma$ with $\langle\chi\rangle \equiv f_\sigma \exp\left(-\frac{\langle\sigma\rangle}{f_\sigma}\right)$,

$$\boxed{\mathcal{L}' = \frac{f_\sigma^2 - f_\pi^2}{\langle\chi\rangle^{*\,2}}\mathcal{A}(\bar\sigma,\ \bar\pi) + \frac{g_A - g_{V\rho}}{\langle\chi\rangle^{*\,2}}\mathcal{B}(\bar\sigma,\ \bar\pi,\ N)}$$

in nuclear matter

where σ and π are redefined as $\sigma \equiv \frac{\langle\chi\rangle}{f_\sigma}\sigma$ and $\pi^a \equiv \frac{\langle\chi\rangle}{f_\sigma}\pi^a$ to get the kinetic term in the form as

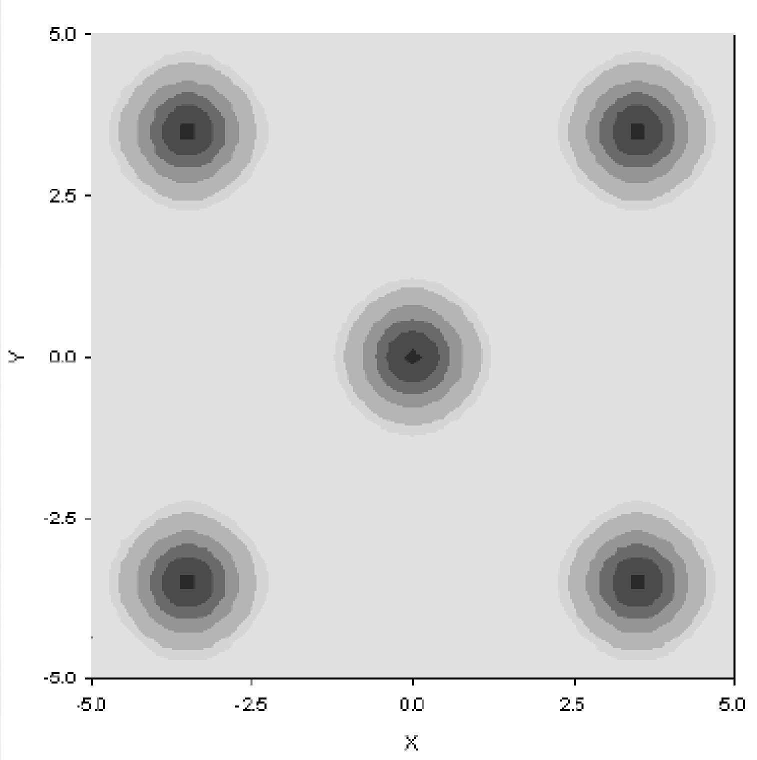
$$\mathcal{L}_{\rm scalar} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a\,.$$

$$\boxed{\frac{m_N^*}{m_N}} \approx \frac{g_V}{g_V^*}\frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \left(\frac{m_\pi^*}{m_\pi}\right)^2 \approx \frac{f_\pi^*}{f_\pi} \boxed{\approx \frac{\langle\chi\rangle^*}{f_\sigma}}$$

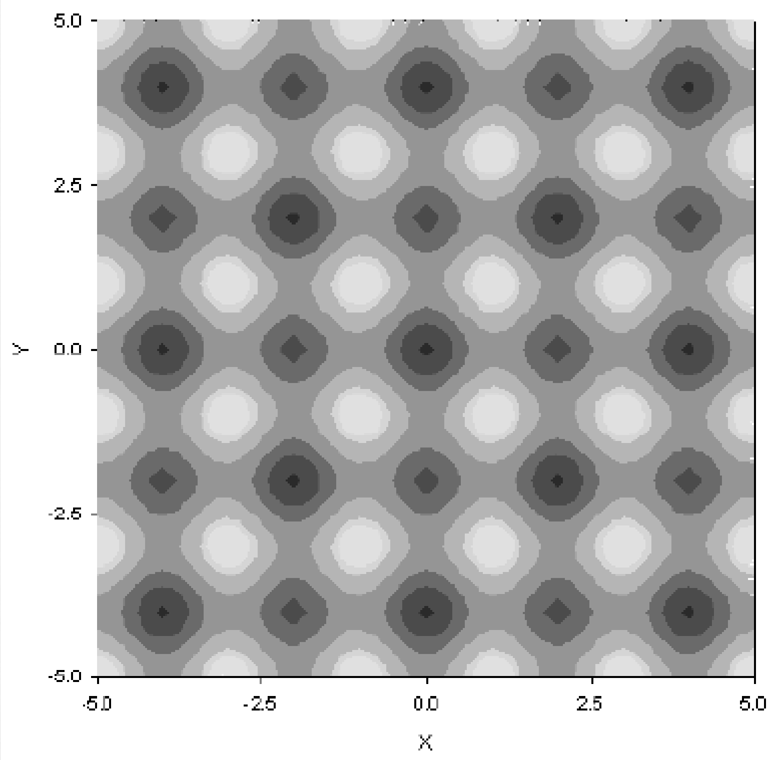
$$\boxed{\langle\theta_\mu^\mu\rangle \propto \langle\chi\rangle^{*\,4}}$$

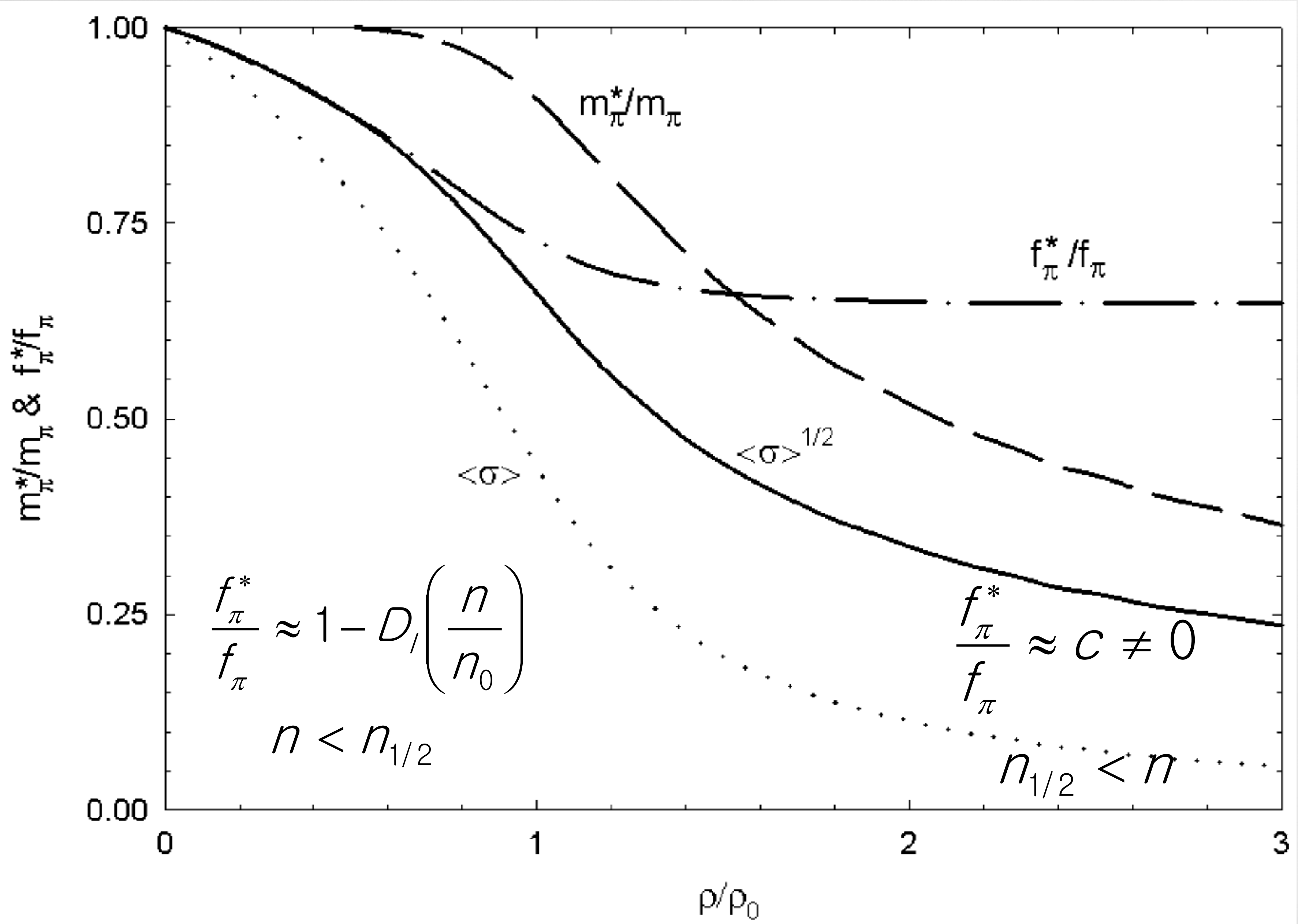
H. J. Lee, B. Y. Park, D. P. Min, M. Rho and V. Vento, ``A Unified approach to high density: Pion fluctuations in skyrmion matter,'' Nucl. Phys. A **723**, 427 (2003)

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{Tr}\left(U^\dagger\partial_\mu U U^\dagger\partial^\mu U\right) + \frac{1}{32e^2}\mathrm{Tr}\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2 + \frac{f_\pi^2 m_\pi^2}{4}\mathrm{Tr}\left(U + U^\dagger - 2\right),$$

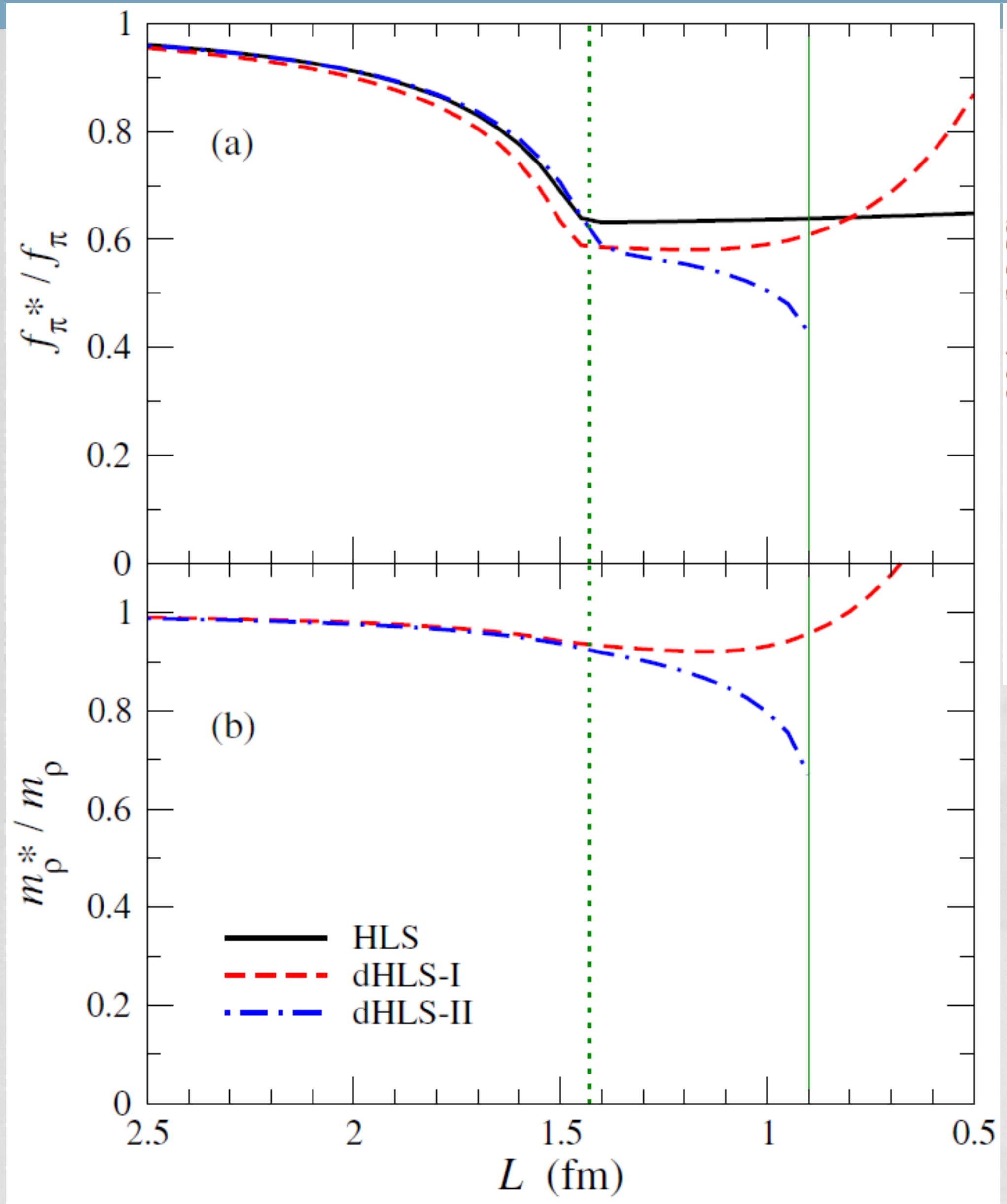


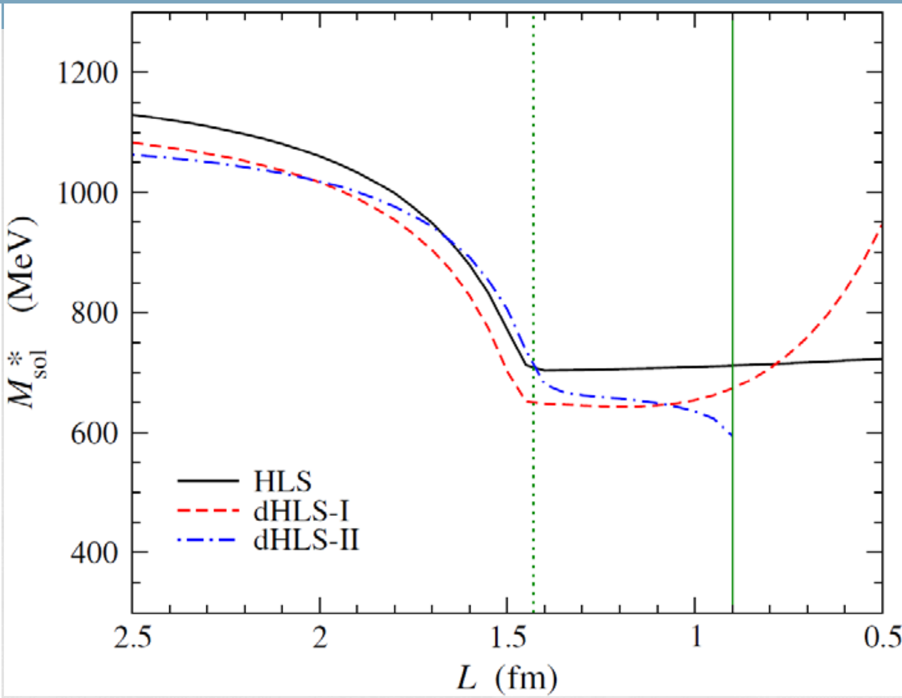
$n = n_{1/2}$





1.00
0.75
0.50
0.25
0.00
0
1
2
3
m^*_π/m_π & f^*_π/f_π
m^*_π/m_π
f^*_π/f_π
<σ>
<σ>$^{1/2}$
ρ/ρ_0
$\frac{f^*_\pi}{f_\pi} \approx 1 - D_l\left(\frac{n}{n_0}\right)$
$n < n_{1/2}$
$\frac{f^*_\pi}{f_\pi} \approx c \neq 0$
$n_{1/2} < n$





Ma, Harada, Lee, Oh, Park, Rho, PRD 90, 034015 (2014)]

$$\frac{m_N^*}{m_N} \approx \frac{g_V}{g_V^*}\frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \left(\frac{m_\pi^*}{m_\pi}\right)^2 \approx \frac{f_\pi^*}{f_\pi} \approx \frac{\langle\chi\rangle^*}{f_\sigma}$$

$\langle\theta_\mu^\mu\rangle \propto \langle\chi\rangle^{*\,4}$ ≈ constant as the nucleon density increases.

$$\langle\theta_\mu^\mu\rangle = \epsilon(n) - 3P(n)$$

$$\frac{\partial}{\partial n}\langle\theta_\nu^\nu\rangle = \frac{\partial\,\epsilon(n)}{\partial n}\left(1 - 3\frac{v_s^2}{c^2}\right), = 0$$

where we used $\frac{v_s^2}{c^2} = \frac{\partial\, P(n)}{\partial n} \Big/ \frac{\partial\,\epsilon(n)}{\partial n}$

$$\boxed{v_s/c = \sqrt{1/3}}$$ in half-skyrmion phase

$$\left.\frac{\partial\,\Omega}{\partial\chi}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle}=0\,,\qquad \left.\frac{\partial\,\Omega}{\partial\omega_0}\right|_{\omega_0=\langle\omega_0\rangle,\,\chi=\langle\chi\rangle}=0\,.$$

They are written as

$$\frac{m_N^2\langle\chi\rangle}{\pi^2 f_\sigma^2}\left[k_F E_F - m_N^{*\,2}\ln\left(\frac{k_F+E_F}{m_N^*}\right)\right] - \frac{f_{\sigma\omega}^2}{f_\sigma^2}g_\omega^2\langle\omega_0\rangle^2\langle\chi\rangle + \left.\frac{\partial\,V(\chi)}{\partial\chi}\right|_{\chi=\langle\chi\rangle}=0\,,$$

$$g_\omega\left(g_{V\omega}-1\right)n - f_{\sigma\omega}^2 g_\omega^2\frac{\langle\chi\rangle^2}{f_\sigma^2}\langle\omega_0\rangle = 0\,.$$

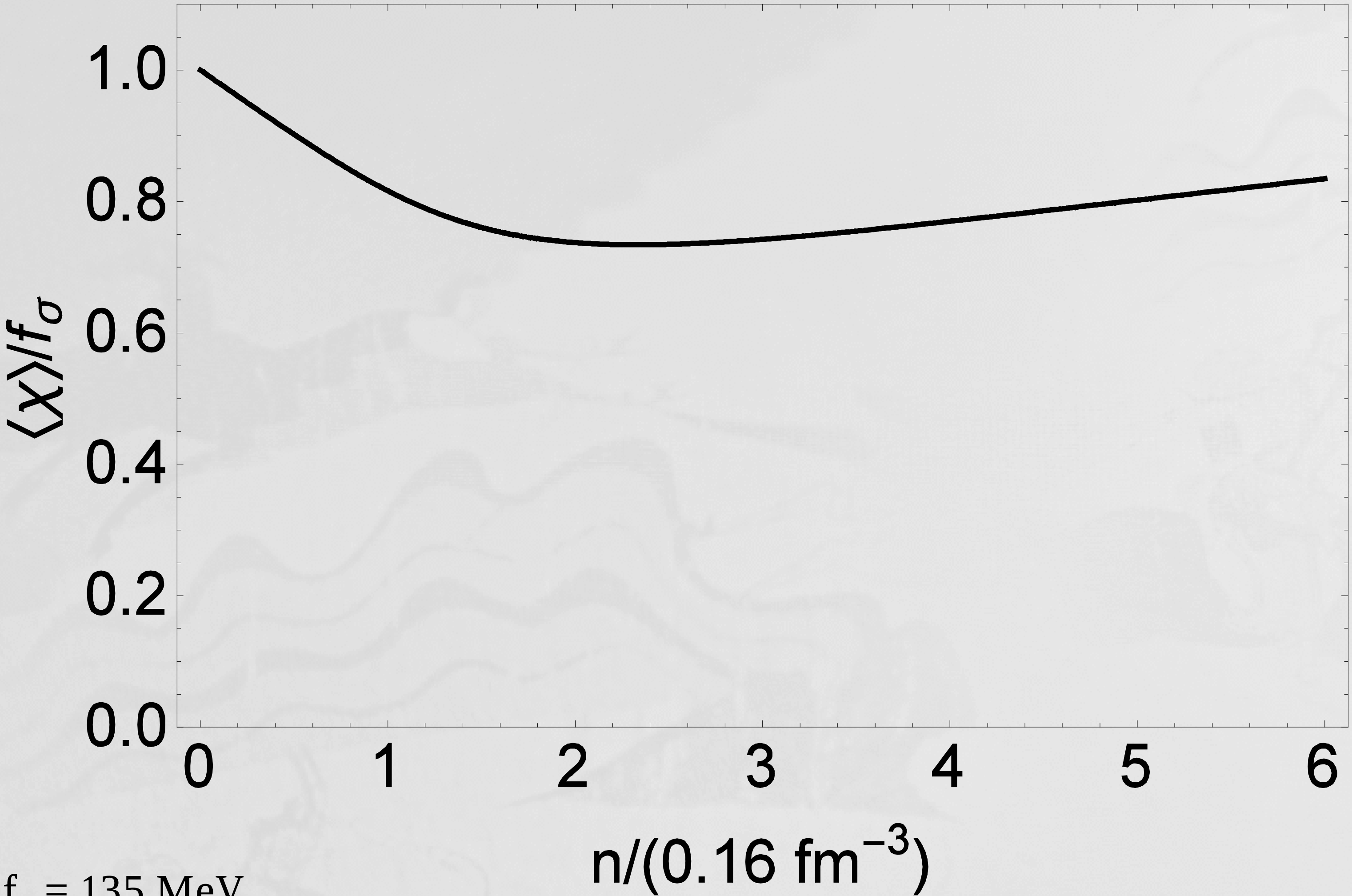
Interestingly, if we calculate

$$\begin{aligned}\langle\theta_\mu^\mu\rangle &= \langle\theta^{00}\rangle - \sum_i\langle\theta^{ii}\rangle\\ &= \epsilon - 3P\end{aligned}$$

, we find that

$$\langle\theta_\mu^\mu\rangle = 4V(\langle\chi\rangle) - \langle\chi\rangle\left.\frac{\partial V(\chi)}{\partial\chi}\right|_{\chi=\langle\chi\rangle}$$ in nuclear matter

$$\theta_\mu^\mu = 4V(\chi) - \chi\frac{\partial V(\chi)}{\partial\chi}$$ in vacuum



f_σ = 135 MeV,
m_σ= 720 MeV,
m_n = 940 MeV,
$g_{\omega NN}$= 6.554,
m_ω = 783 MeV.

$\boxed{\langle \theta^\mu_\mu \rangle \propto \langle \chi \rangle^{*\,4} \text{ is not constant .}}$

Vector Manifestation Fixed Point

- In HLS for mesons[Harada, Yamawaki, Phys. Rept. 381, 1 (2003)], Wilsonian matching of the parameters in HLS with the values in QCD was studied. When the current correlators of HLS and QCD are matched with each other at the matching scale $q^2 = \Lambda^2$, the bare parameters of HLS are changed as

$$g_\rho{}^*(\mu = \Lambda) \to 0,\ a_\rho{}^*(\mu = \Lambda) \to 1 \text{ and } f_\pi{}^*(\mu = \Lambda) \to \sqrt{\frac{N_f \Lambda^2}{2(4\pi)^2}}$$

at $\langle \bar{q} q \rangle^* = 0$, which is called Vector Manifestation(VM) Fixed Point.

- The change of the bare parameters in medium makes the parameters move to

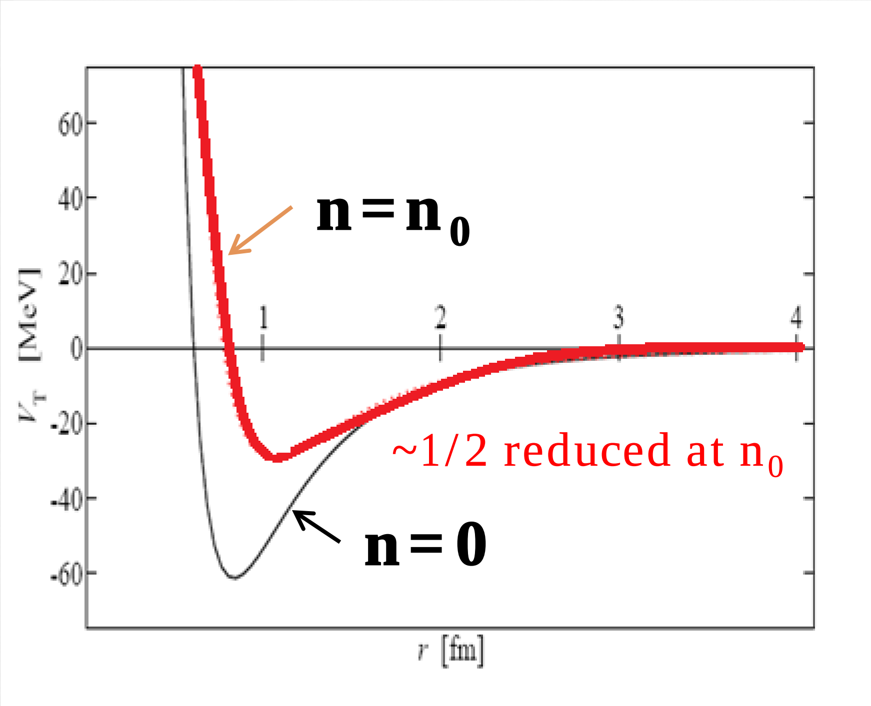
$$g_\rho{}^*(\mu = 0) \to 0 \text{ and } f_\pi{}^*(\mu = 0) \to 0$$

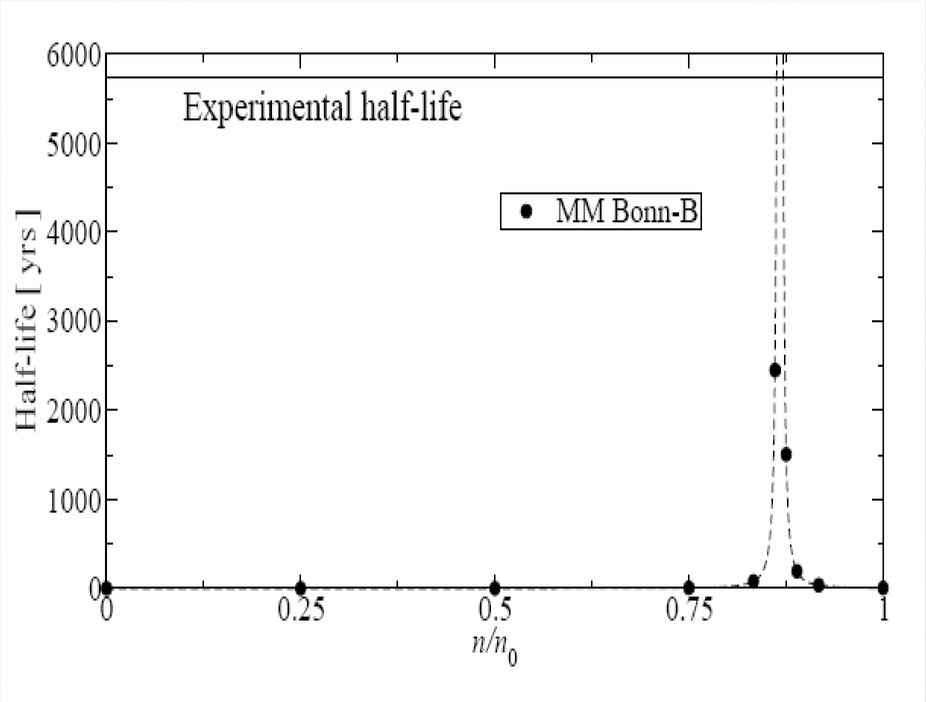
by RGE, where we used, by the RGE of $f_\pi{}^*(\mu)$,

$$F_\pi^2(0) = F_\pi^2(\Lambda) - \frac{N_f \Lambda^2}{2(4\pi)^2}$$

C14 dating probes scaling

J.W. Holt et al, PRL **100**, 062501 (08)

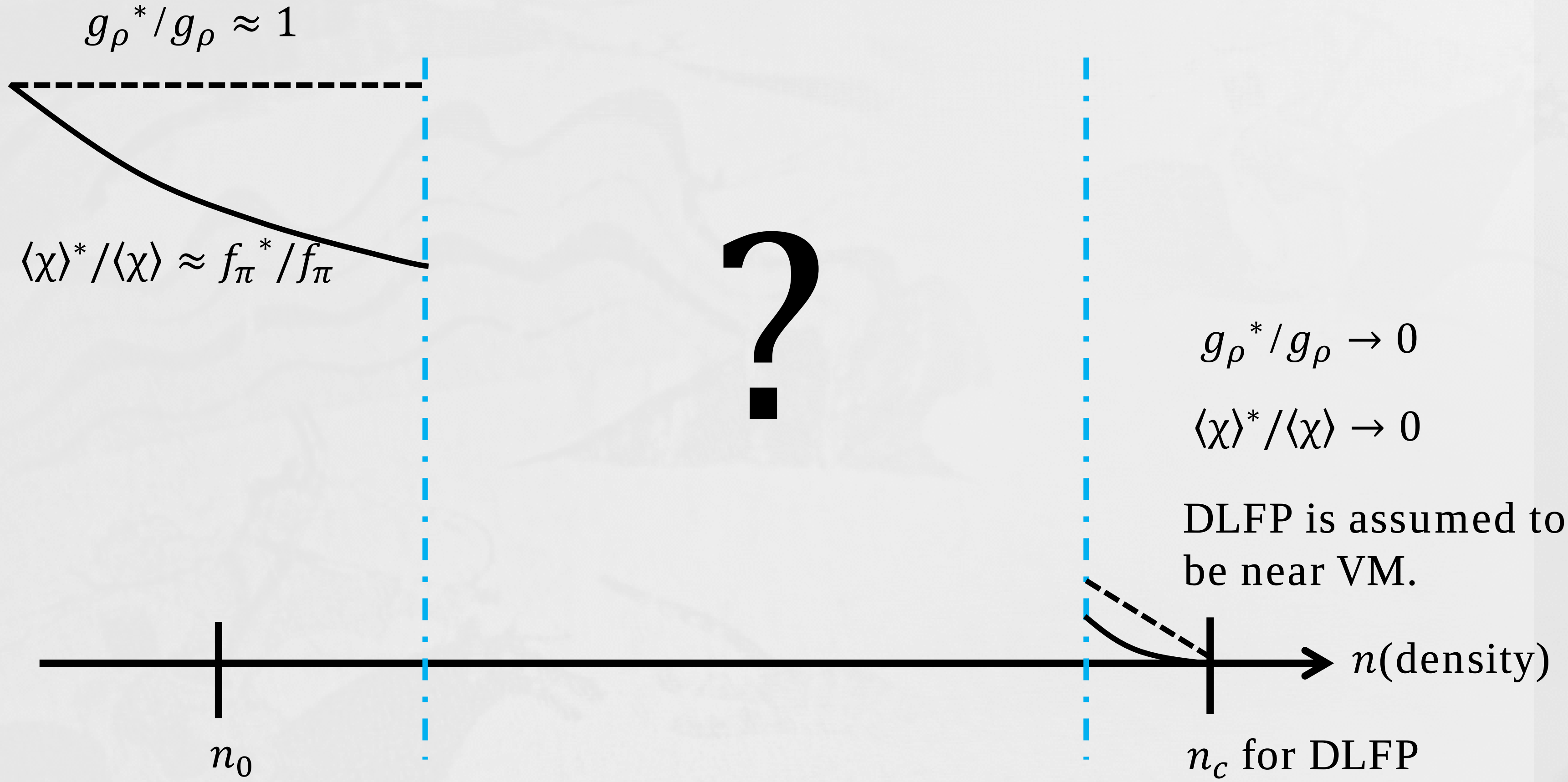




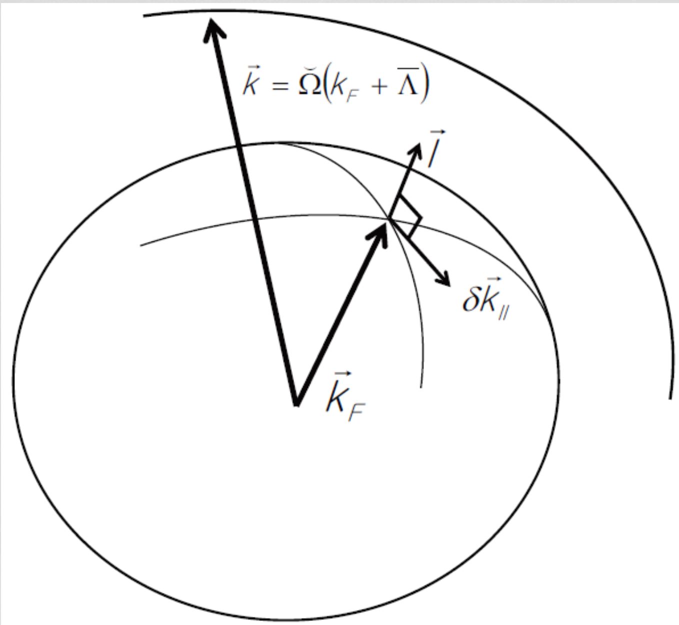
For the reduction of the tensor force, we need to have $g_\rho{}^*/g_\rho \approx 1$

What we know from the phenomenology

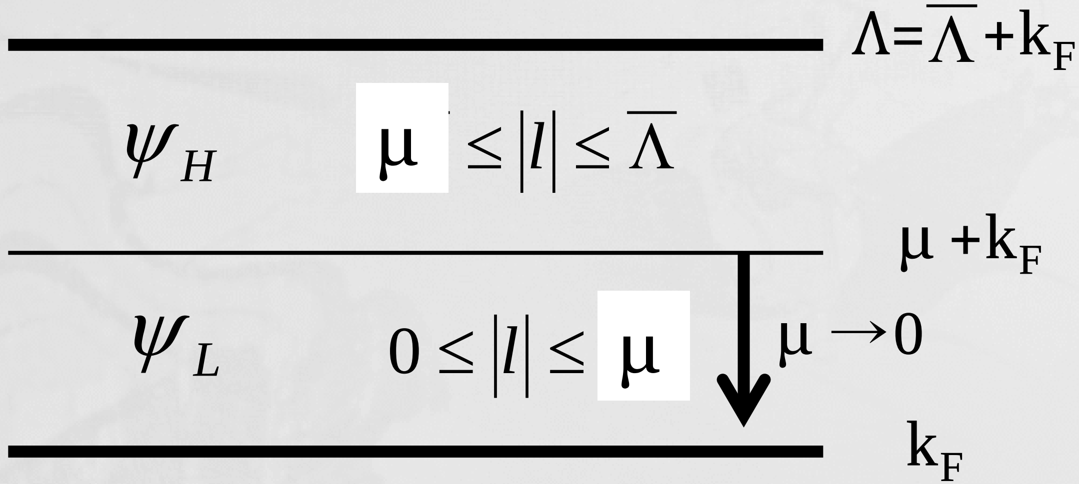
Relying on phenomenology[PRL**100** 062501 (2008) for C14 decay, Prog. Part. Nucl. Phys. 52, 85 (2004) for deeply bound pionic systems], we could find roughly the density dependence of $\langle\chi\rangle^*$ and $g_\rho{}^*$ near the saturation density n_0 and the density where DLFP sets on.



Wilsonian RG flow in Fermi-liquid(medium)



The momentum of the fermion $\vec{k}$ is given by $|\vec{k}| \le \Lambda$, where Λ is the cutoff of the bare model. If we define $l \equiv |\vec{k}| - k_F$, $|l| \le \bar{\Lambda}$, where $\bar{\Lambda} \equiv \Lambda - k_F$.



The RG flow of the renormalized parameter will also depend on a density ,

$$\beta\left(g^i_{eff}(\mu, k_F)\right) = \mu \frac{d}{d\mu} g^i_{eff}(\mu, k_F)$$

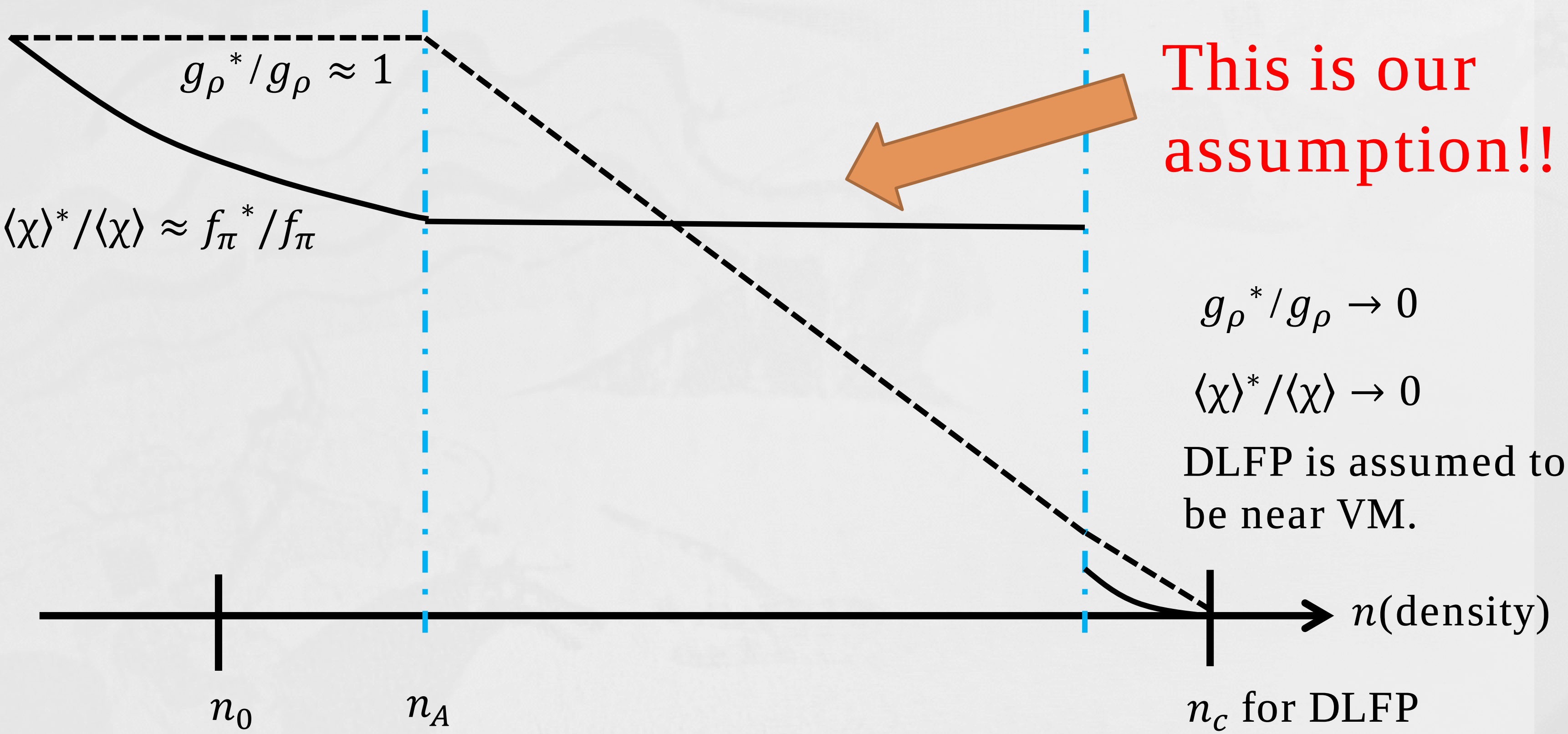
Wilsonian RG in Fermi-Liquid

On the fermi-liquid fixed point, the parameters should be closely related to each other with increasing density. That is shown by

$$\text{from } m^*(k_{F1}, \mu = 0) \text{ to } m^*(k_{F2}, \mu = 0) \text{ to preserve}$$

$$\beta(k_{F1}, \mu = 0) = \beta(k_{F2}, \mu = 0) = 0.$$

The gap equations should be modified by the loop contributions dependent of g_ρ^* and g_ω^*.



If our assumption is correct, then, a question follows.

"Could we find the equation of state(EoS) for the nuclear matter describing $v_s = c/\sqrt{3}$ by considering the density dependence of the parameters satisfying our assumption?"

We answer to that question by using the $V_{\text{low k}}$ applied to the dense nuclear matter.

- The leading order terms in our model(bsHLS) generates one boson exchange potential shown by

$$\mathcal{L}_{\text{Bonn A}} = \frac{g_{\pi NN}}{m_N}\bar{N}\gamma^\mu\gamma^5\partial_\mu\vec{\pi}\cdot\frac{\vec{\tau}}{2}N + g_{\sigma NN}\bar{N}\sigma N$$
$$-g_{VNN}\bar{N}\gamma^\mu V_\mu N - \frac{f_v}{4m_N}\bar{N}\sigma^{\mu\nu}\left(\partial_\mu V_\nu - \partial_\nu V_\mu\right)N\,,$$

where $V_\mu = \vec{\rho}_\mu\cdot\frac{\vec{\tau}}{2}$ or $\frac{\omega_\mu}{2}$ and g_{VNN}, f_V are proportional to the hidden gauge coupling g_V.

- Here, the tensor force arises by the loop calculation as

$$\Gamma_{VNN}(q^2) = A(q^2)\gamma_\mu + B(q^2)\sigma_{\mu\nu}q^\nu.$$

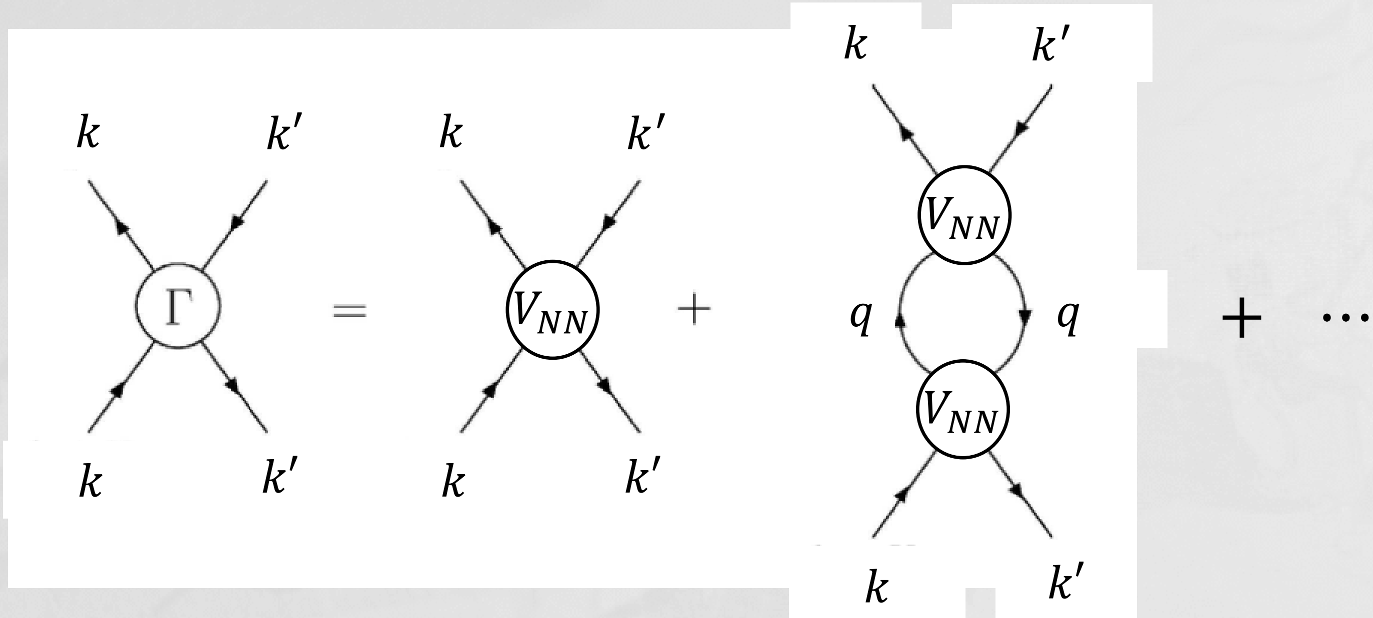
- And, in the momentum space,

$$V_M^t = S_M\frac{g^2_{MNN}}{12m_N}\frac{S_{12}(\vec{q})}{|\vec{q}|^2+m_M^2}$$

in the momentum space, where $M=\pi$, ρ, $S_{\pi(\rho)} = -1(+1)$ and $S_{12}(\vec{q}) = 3\left(\vec{\sigma}_1\cdot\vec{q}\right)\left(\vec{\sigma}_1\cdot\vec{q}\right) - \vec{\sigma}_1\cdot\vec{\sigma}_2\,|\vec{q}|^2$. πNN interaction mainly contributes to NN interaction through the tensor force.

- In nuclear matter, the density dependence of the parameters obtained in bsHLS for the mass and coupling constants are used for π, ρ, ω, σ as $\frac{m^*}{m}=\frac{\langle\chi\rangle^*}{f_\sigma}$ and $\frac{g^*}{g}$.

The Bonn A potential expressed by our theory is renormalized to get $V_{\text{low k}}$ potential having the density dependent parameters.

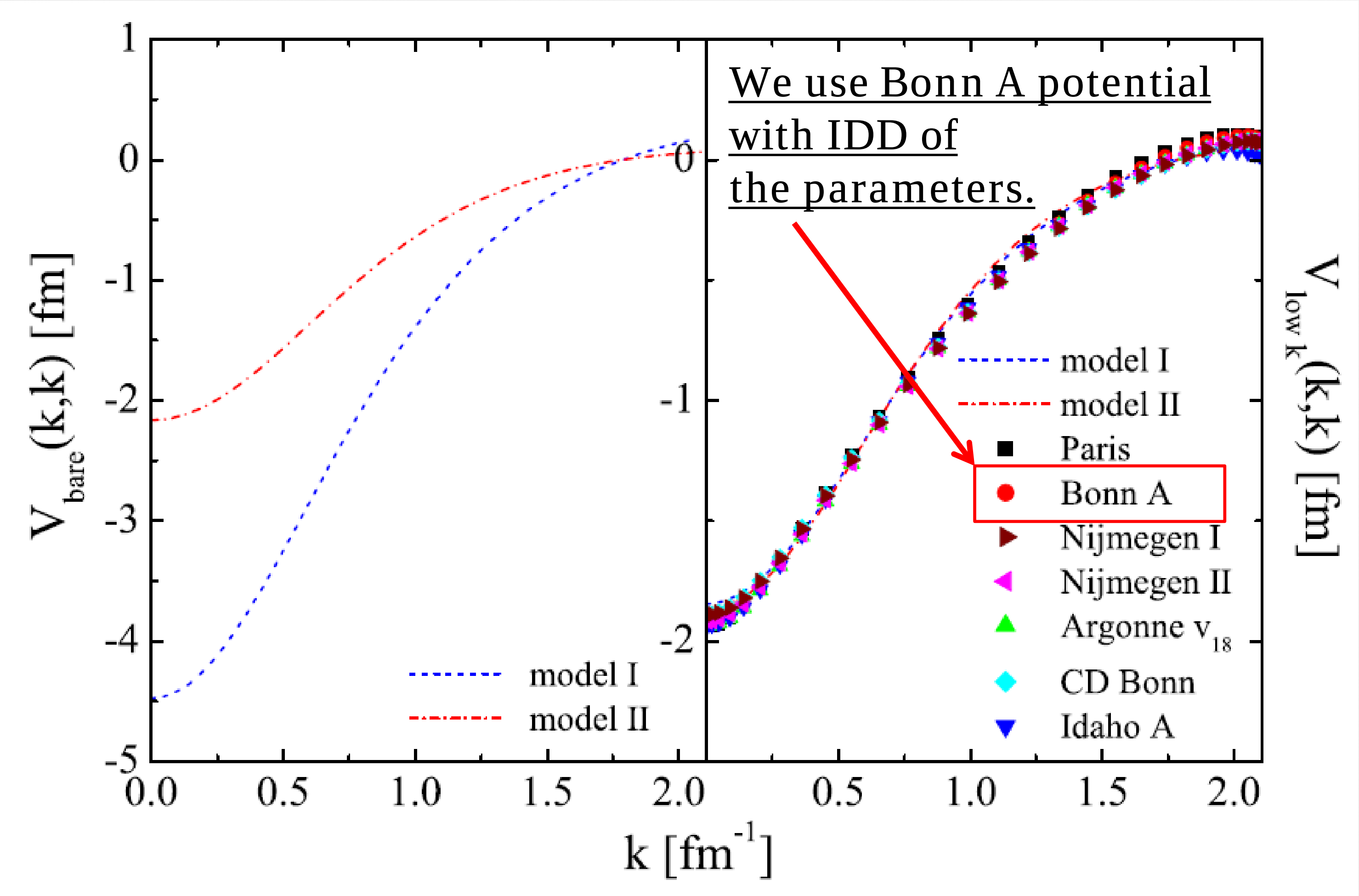


$$T(k',k,k^2) = V_{\rm NN}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{V_{\rm NN}(k',q)T(q,k,k^2)}{k^2-q^2}q^2dq,$$

$$T_{\rm lowk}(k',k,k^2) = V_{\rm lowk}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^\Lambda \frac{V_{\rm lowk}(k',q)T_{\rm lowk}(q,k,k^2)}{k^2-q^2}q^2dq,$$

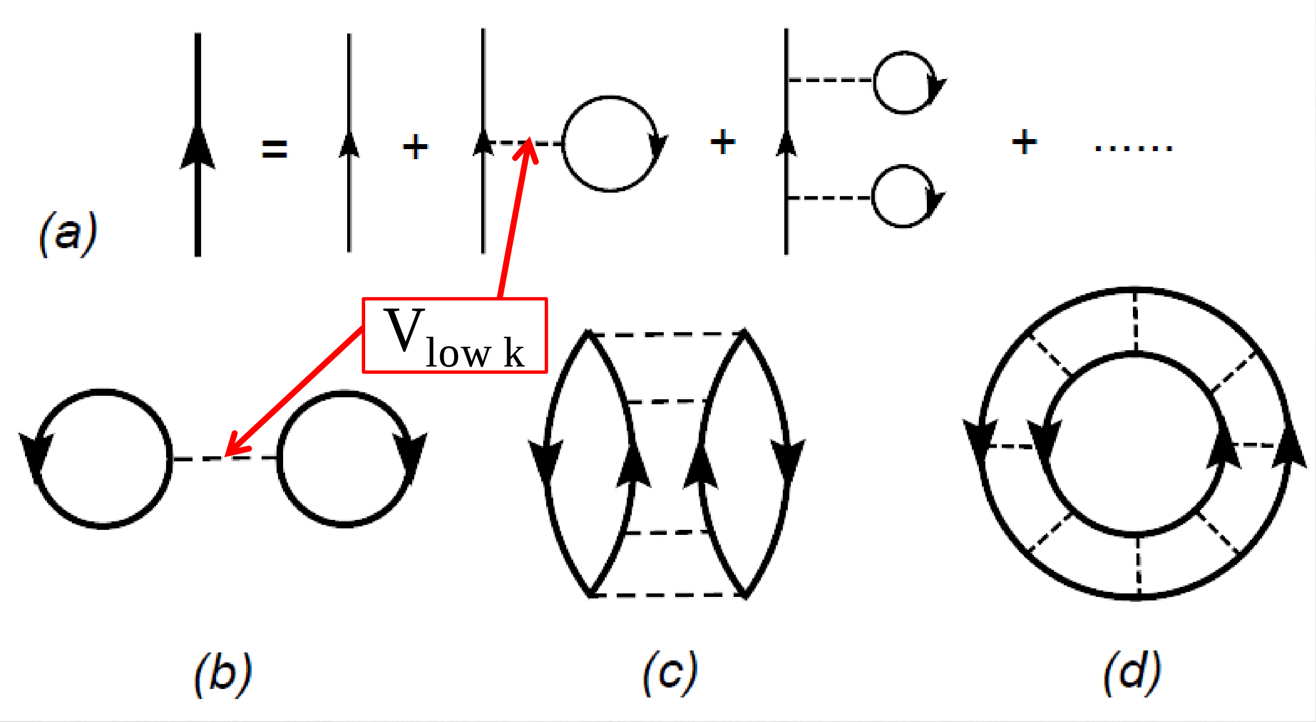
$$T(k',k,k^2) = T_{lowk}(k',k,k^2);\,(k',k) \leq \Lambda.$$

- The model-independent low momentum interaction, called V_{lowk}, is obtained, which reproduces the same phase shifts for the NN scattering data and deuteron pole which are the inputs.



S. K. Bogner, T. T. S. Kuo and A. Schwenk, Phys. Rept. 386, 1 (2003)

- We obtain the ground state energy of the symmetric nuclear matter and pure neutron matter by calculating the single particle energy for the diagram (a) and the pphh ring diagrams for the diagram (b), (c) and (d) summed to all orders within a model space of the cutoff Λ.



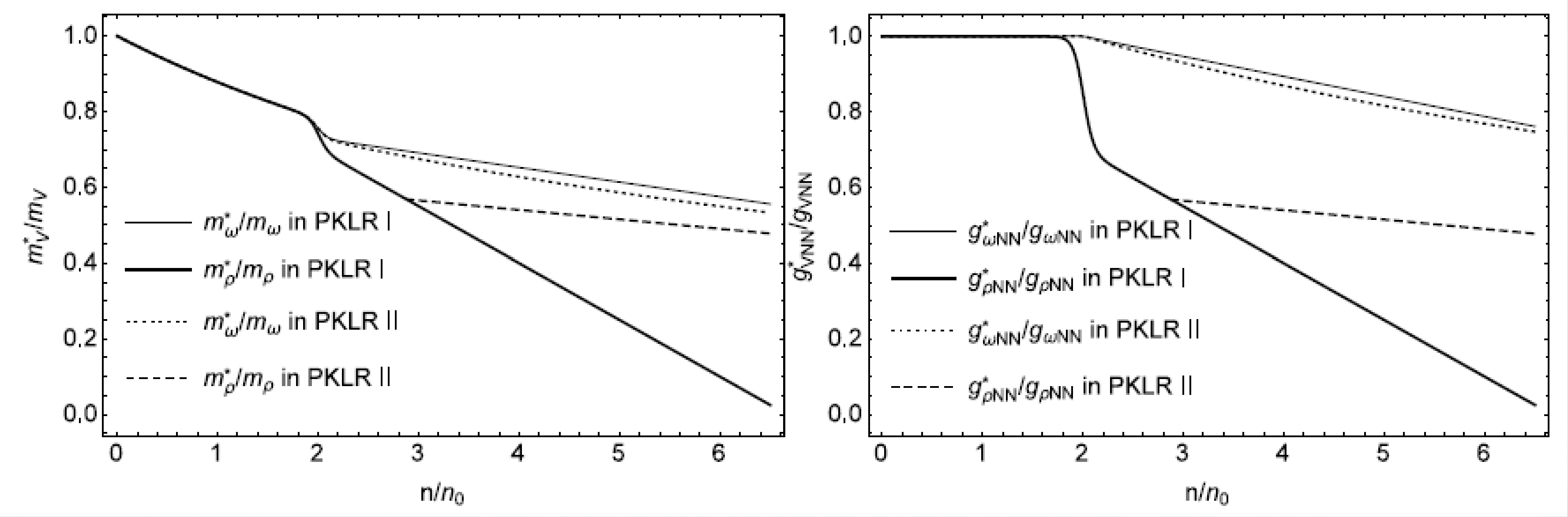
As expected in the mean field calculation, we consider the decreasing $\langle\chi\rangle^*$ in $n < n_{1/2}$ in a simple form as

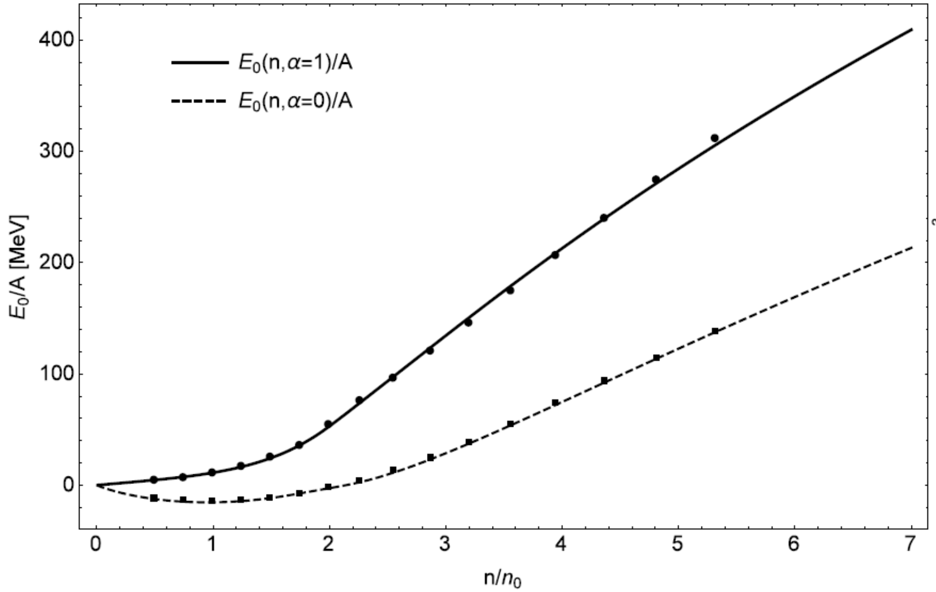
$$\frac{\langle\chi\rangle^*}{f_\sigma} = \frac{1}{1 + c_I\ (n/n_0)}$$

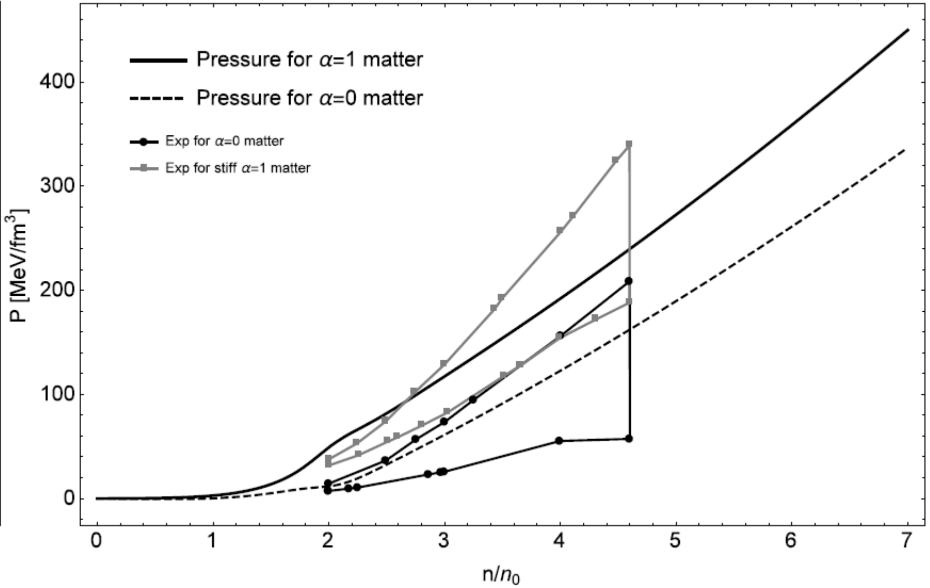
and $\langle\chi\rangle^*$ stays constant in $n_{1/2} < n$ as

$$\frac{\langle\chi\rangle^*}{f_\sigma} = \kappa \equiv \frac{1}{1 + c_I\ (n_{1/2}/n_0)}\,.$$

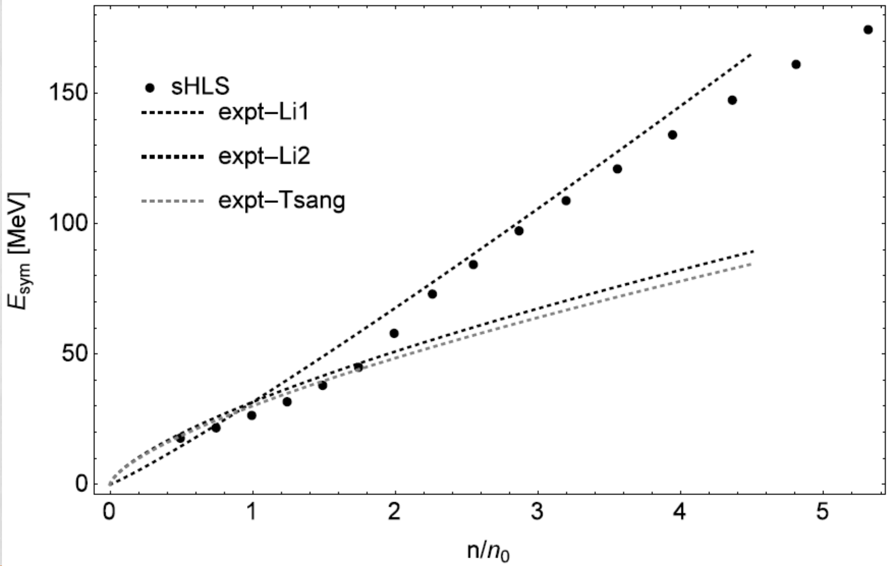
$$\frac{m_N^*}{m_N} \approx \frac{g_V}{g_V^*}\frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \left(\frac{m_\pi^*}{m_\pi}\right)^2 \approx \frac{f_\pi^*}{f_\pi} \approx \frac{\langle\chi\rangle^*}{f_\sigma}$$

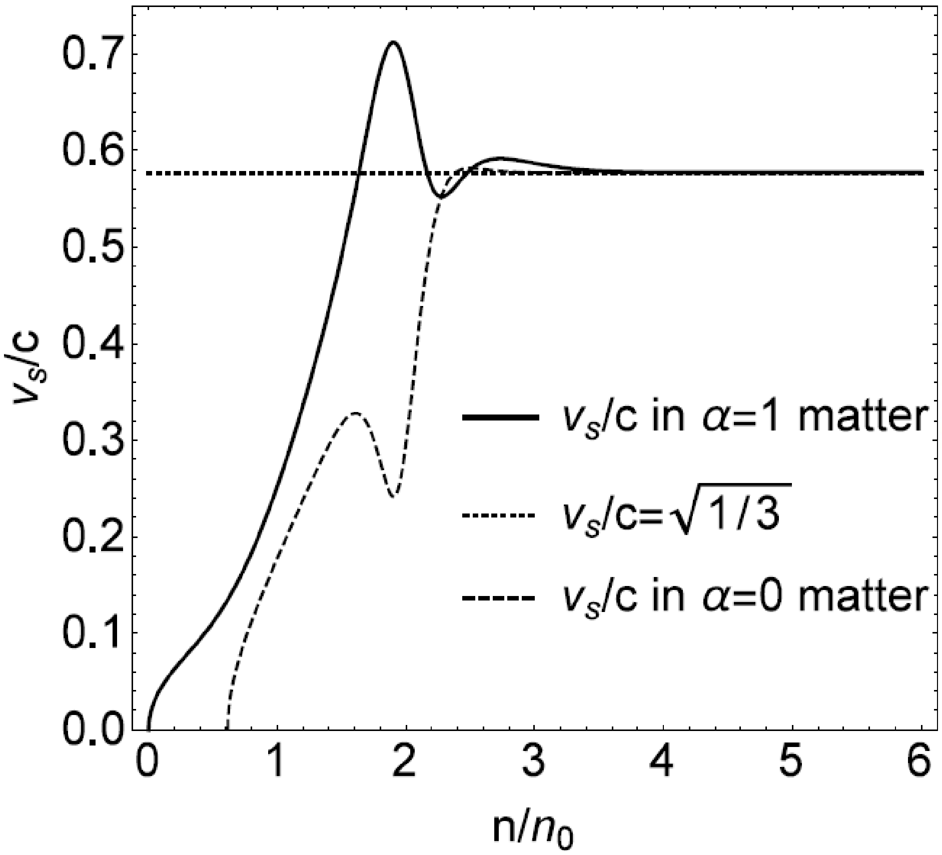


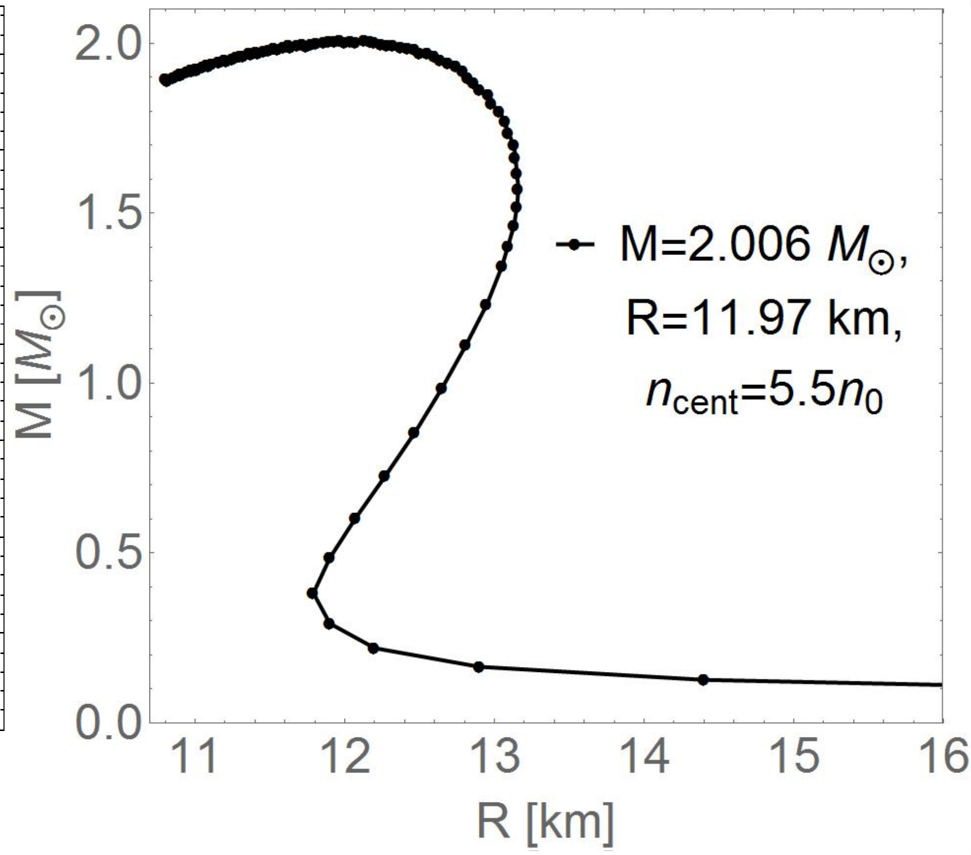




$E_0/A = -m_N + B\left(\frac{n}{n_0}\right)^{1/3} + D\left(\frac{n}{n_0}\right)^{-1}$ in $n_{1/2} < n$, derived by the condition $v_s^2/c^2 = 1/3$







$$\langle \theta^\mu_\mu \rangle = \epsilon(n) - 3P(n) = \text{constant} \Rightarrow \boldsymbol{v_s = c/\sqrt{3}}$$

The answer: Yes, we can find the EoS for the nuclear matter describing $\boldsymbol{v_s = c/\sqrt{3}}$ by considering the density dependence of the parameters satisfying our assumption.

The EoS predicts $M_{\max} = 2.006 M_\odot$.

Summary

- sHLS theory has three following predictions by the interplay between the hidden scale symmetry ($\langle\chi\rangle^*$)and the hidden local symmetry($g_{\rho,\omega}{}^*$) on the fermi-liquid fixed point with increasing density.

(1) Parity doublet structure arises in the half-skyrmion phase: $m_N{}^* \sim \langle\chi\rangle^* \sim m_0$, constant as a density increases. The magnitude of m_0 affects the value of the compression modulus so that it is an important parameter in the nuclear structure.

(2) The slop of the symmetry energy is changed from soft to stiff at $n \sim n_{1/2}$, which may be confirmed in the heavy ion experiment through the transport model.

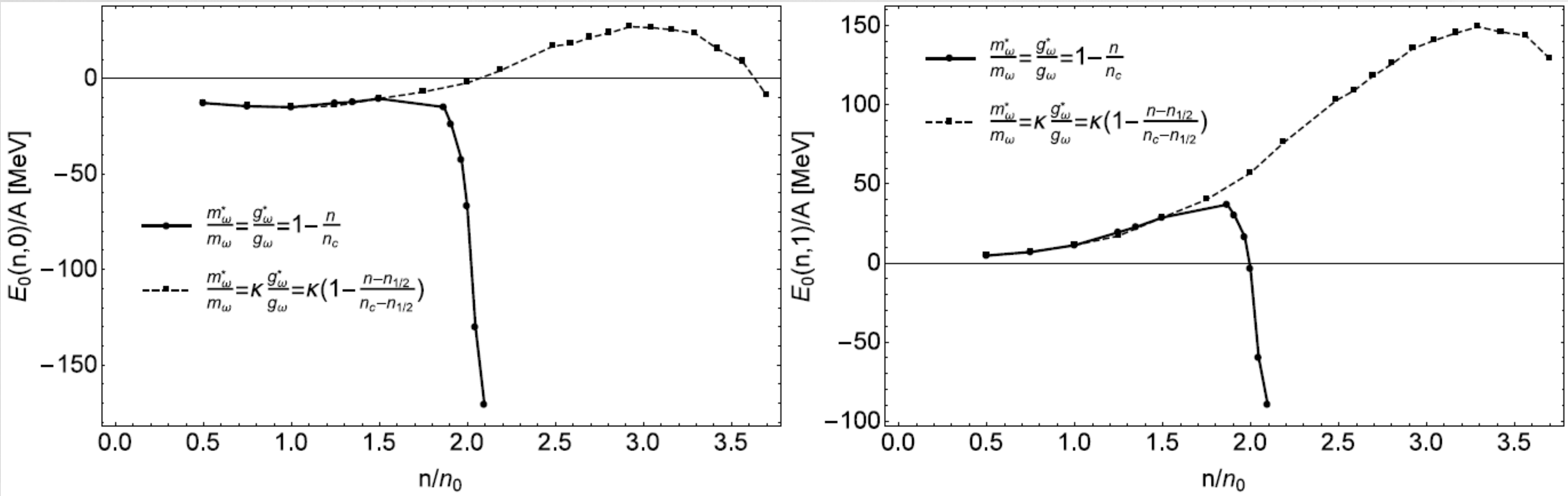
(3) The sound velocity in the nuclear matter is predicted as $\boldsymbol{v_s = c/\sqrt{3}}$ which constrain the EoS of nuclear matter providing $M_{\max} = 2.006 M_{\odot}$. The EoS describing the neutron stars can gives the prediction which could be tested by Gravitational Wave detection.

We are addressing the question, whether the scale symmetry could be emergent in dense nuclear matter. We expect that Nature gives the answer.

Thank you

Back up

U(2) symmetry for ρ and ω is broken in dense nuclear matter



$$m^*{}_\omega/m_\omega \approx g^*{}_\omega/g_\omega \approx g^*{}_\rho/g_\rho \approx (1-n/n_c)$$

$$m^*{}_\omega/m_\omega \approx \frac{\langle\chi\rangle}{f_{0\sigma}} g^*{}_\omega/g_\omega \approx \frac{\langle\chi\rangle}{f_{0\sigma}} g^*{}_\rho/g_\rho \approx \frac{\langle\chi\rangle}{f_{0\sigma}}(1-n/n_c)$$

Pion tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^{*\,2}}{4\pi} \tau_1 \tau_2 S_{12} \mathcal{I}(m_M^* r)$$

$$\mathcal{I}(m_M^* r) \equiv m_M^* \left(\left[\frac{1}{(m_M^* r)^3} + \frac{1}{(m_M^* r)^2} + \frac{1}{3 m_M^* r} \right] e^{-m_M^* r} \right)$$

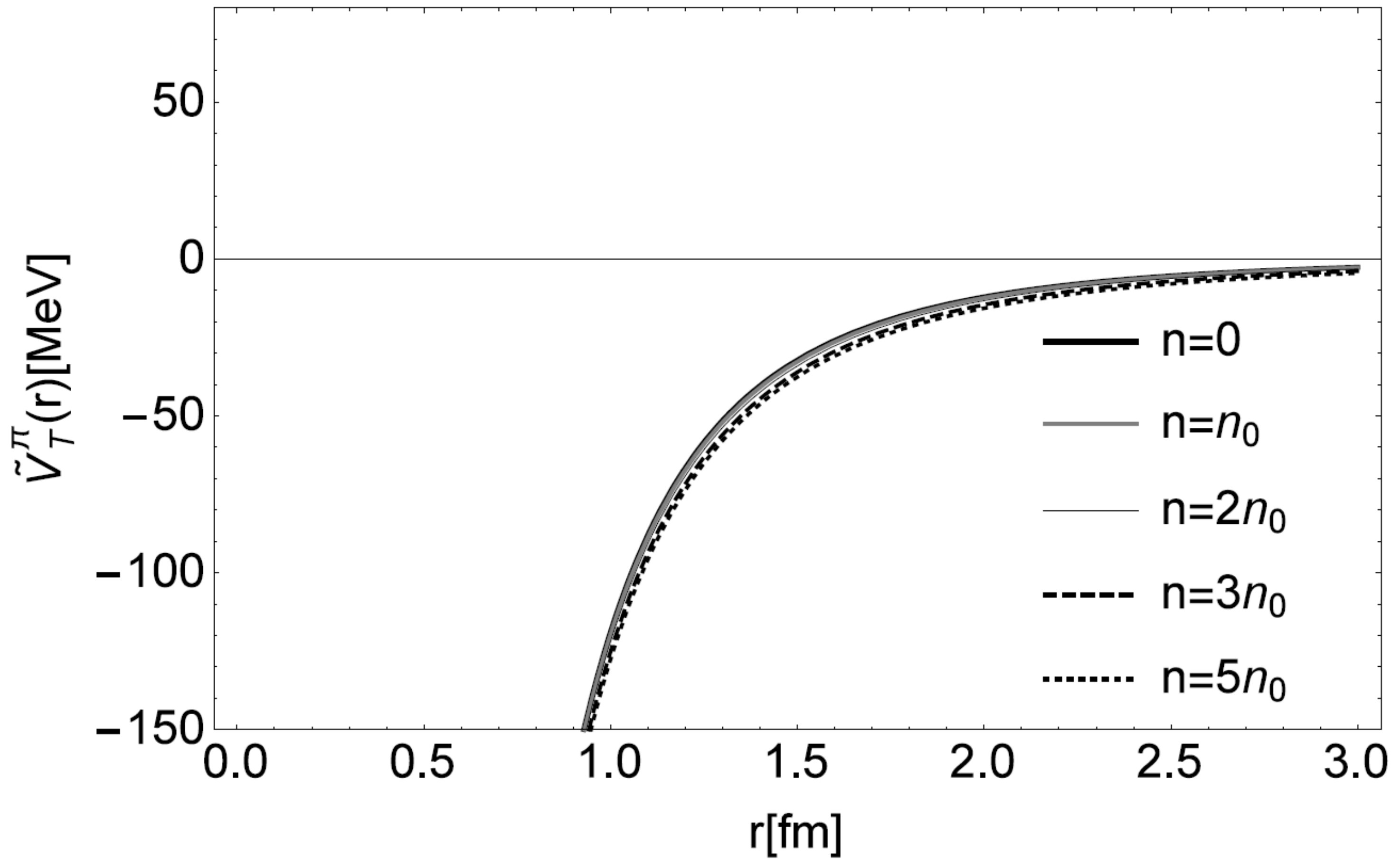
where $M = \pi,\ \rho,\ S_{\rho(\pi)} = +1(-1)$ and

$$S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\begin{aligned} R_\pi &\approx \frac{g^*_{\pi NN}}{g_{\pi NN}} \frac{m_N}{m_N^*} \frac{m_\pi^*}{m_\pi} \\ &\approx \begin{cases} \Phi_I \times \Phi_I^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-I} \\ \kappa \times \kappa^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-II} \end{cases} \end{aligned}$$

$$\begin{aligned} & f_\pi^{*\,2} m_\pi^{*\,2} = m_q \langle \bar{q} q \rangle + \sum_{n>1} c_n \langle (\bar{q} q)^n \rangle \\ \Rightarrow \quad & \kappa^2 m_\pi^{*\,2} = \sum_{n>1} c_n \langle (\bar{q} q)^n \rangle \,. \end{aligned}$$

$$\frac{m_\pi^*}{m_\pi} = \left(\frac{1}{1+0.13*n/n_0}\right)^{1/2} \frac{1}{1+\exp\left(\frac{n-n_{1/2}}{0.05n_0}\right)} + \left(1-0.15*\frac{n}{n_0}\right)\frac{1}{1+\exp\left(-\frac{n-n_{1/2}}{0.05n_0}\right)}$$



Rho tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^{*2}}{4\pi} \tau_1 \tau_2 S_{12} \mathcal{I}(m_M^* r)$$

$$\mathcal{I}(m_M^* r) \equiv m_M^* \left(\left[\frac{1}{(m_M^* r)^3} + \frac{1}{(m_M^* r)^2} + \frac{1}{3 m_M^* r} \right] e^{-m_M^* r} \right)$$

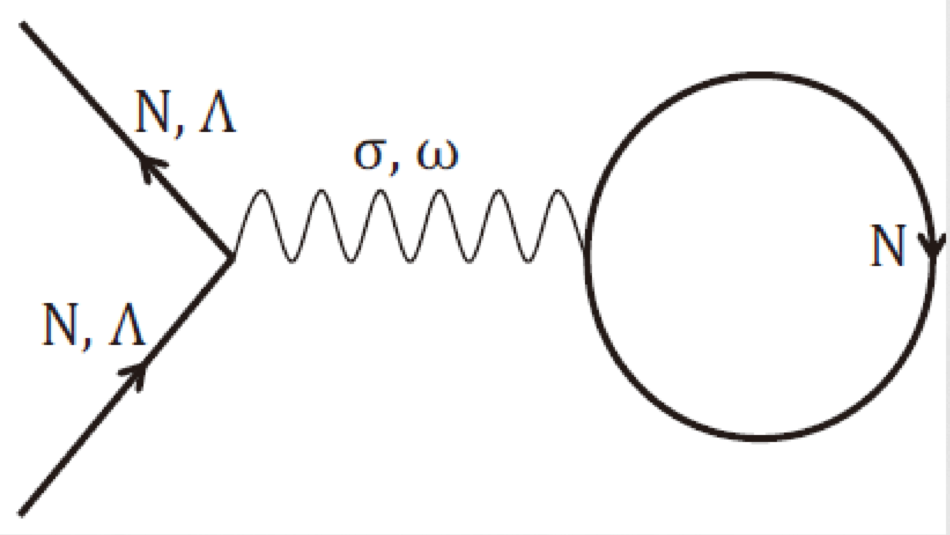
where $M = \pi, \rho$, $S_{\rho(\pi)} = +1(-1)$ and

$$S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$\begin{aligned} R_\rho &\approx \frac{g_{\rho NN}^*}{g_{\rho NN}} \frac{m_N}{m_N^*} \frac{m_\rho^*}{m_\rho} \\ &\approx \left(\frac{g^*}{g}\right)^2 \\ &\approx \begin{cases} 1 & \text{for R-I} \\ \Phi_{II}^2 & \text{for R-II} \end{cases} \end{aligned}$$

$$\begin{aligned} m_\rho^*/m_\rho &\approx \left(\frac{g^*}{g}\right)\left(\frac{f_\pi^*}{f_\pi}\right) \\ &\approx \begin{cases} \Phi_I & \text{for R-I} \\ \Phi_{II} \times \kappa & \text{for R-II} \end{cases} \end{aligned}$$

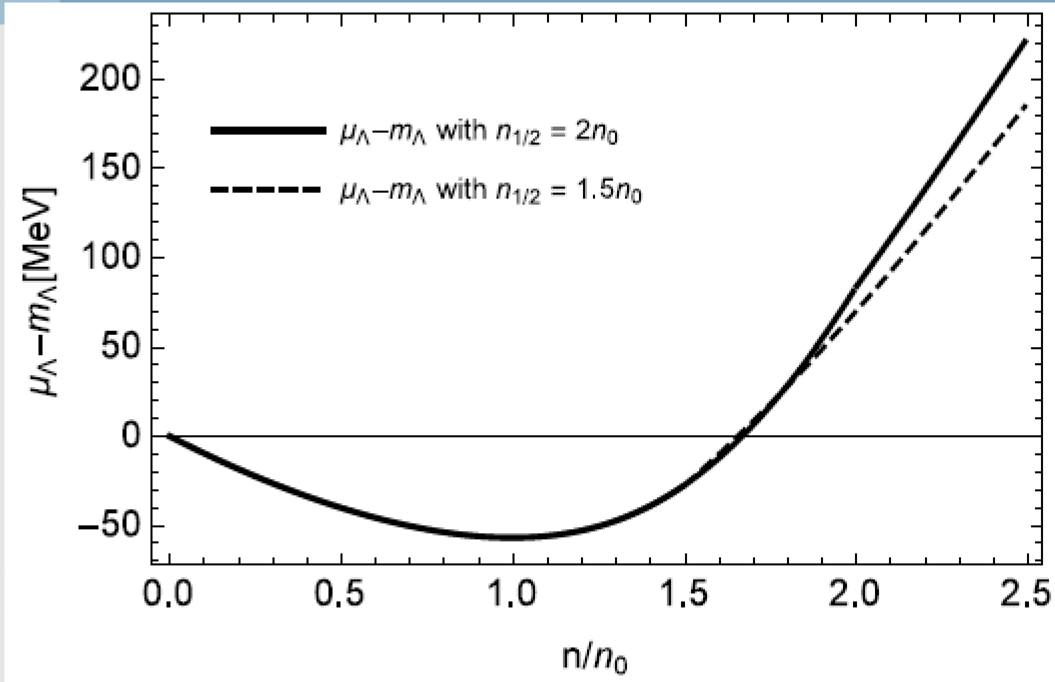
Hyperon in the nuclear matter



$$\mu_\Lambda = m_\Lambda^* - \frac{g_{\sigma\Lambda}^* g_{\sigma N}^*}{m_\sigma^{*\,2}} n_s + \frac{g_{\omega\Lambda}^* g_{\omega N}^*}{m_\omega^{*\,2}} n$$

$$= m_\Lambda^* + \frac{2}{3}\left(-\frac{g_{\sigma N}^{*\,2}}{m_\sigma^{*\,2}} n_s + \frac{g_{\omega N}^{*\,2}}{m_\omega^{*\,2}} n\right)$$

$$g_{\sigma\Lambda} \approx \tfrac{2}{3} g_{\sigma N} \text{ and } g_{\omega\Lambda} \approx \tfrac{2}{3} g_{\omega N}$$



	Parameters for Λ Mass Shift
R-I	$\frac{m^*_\Lambda}{m_\Lambda} = \frac{m^*_N}{m_N} = \frac{m^*_\sigma}{m_\sigma} = \frac{m^*_\omega}{m_\omega} = \frac{1}{1+c_I*\frac{n}{n_0}}$ $\frac{g^*_{\omega\Lambda}}{g_{\omega\Lambda}} = \frac{g^*_{\omega N}}{g_{\omega N}} = \frac{g^*_\omega}{g_\omega} = \frac{g^*_{\sigma\Lambda}}{g_{\sigma\Lambda}} = \frac{g^*_{\sigma N}}{g_{\sigma N}} = 1$
R-II	$\frac{m^*_\Lambda}{m_\Lambda} = \frac{m^*_N}{m_N} = \frac{m^*_\sigma}{m_\sigma} = \kappa = \frac{1}{1+c_I*\frac{n_{1/2}}{n_0}}$ $\frac{m^*_\omega}{m_\omega} = \kappa\frac{g^*_\omega}{g_\omega}$ & $\frac{g^*_{\omega\Lambda}}{g_{\omega\Lambda}} = \frac{g^*_{\omega N}}{g_{\omega N}} = \frac{g^*_\omega}{g_\omega}$ $\frac{g^*_{\sigma\Lambda}}{g_{\sigma\Lambda}} = \frac{g^*_{\sigma N}}{g_{\sigma N}} = 1$

Table 4: The "bare" parameter scaling for mean-field estimate of Λ mass shift in dense matter. The only scaling parameter is chosen to be $c_I = 0.13$ as in Section 5. The vacuum scalar (dilaton) mass is taken to be $m_\sigma = 720\,\mathrm{MeV}$ so as to give $\sim$ 600 MeV at nuclear matter density appropriate for RMF approach. We have taken $\frac{3}{2}g_{\omega\Lambda} = g_{\omega N} = 12.5$ and $\frac{3}{2}g_{\sigma\Lambda} = g_{\sigma N} = m_N/f_\pi$. The empirical values $m_N = 939\,\mathrm{MeV}$, $m_\Lambda = 1116\,\mathrm{MeV}$ and $m_\omega = 783\,\mathrm{MeV}$ are taken from the particle data booklet. The scaling $\frac{g^*_\omega}{g_\omega} \approx (1 - 0.053(n - n_{/2})/n_0)$ is taken as the "best fit" from the analysis in Section 5.

The density where $\mu_\Lambda - m_\Lambda$ becomes positive is in the range of BS constraint.[P. F. Bedaque and A. W. Steiner, "Hypernuclei and the hyperon problem in neutron stars," arXiv:1412.8686 [nucl-th].]

The notation of the particles

Coupling	*Bosons (Strength of Coupling)* $I = 0$ $[1]$	$I = 1$ $[\tau_1 \cdot \tau_2]$	*Central* $[1]$	*Characteristics of Predicted Forces* *Spin-Spin* $[\sigma_1 \cdot \sigma_2]$	*Tensor* $[S_{12}]$	*Spin-Orbit* $[L \cdot S]$
ps	η (weak)	π (strong)	—	**weak,** coherent with *v*, *t*	**strong**	—
s	σ (strong)	δ (weak)	**strong, attractive**	—	—	coherent with *v*
v	ω (strong)	ρ (weak)	**strong, repulsive**	weak coherent with *ps*	opposite to *ps*	**strong,** coherent with *s*
t	ω (weak)	ρ (strong)	—	**weak,** coherent with *ps*	opposite to *ps*	—

I denotes the isospin of a boson. The characteristics quoted refer to $I = 0$ bosons (no isospin dependence). The isovector ($I = 1$) boson contributions, carrying a factor $\tau_1 \cdot \tau_2$, provide the isospin-dependent forces.

The parameters for Bonn A potential

		Potential A	
	m_α (MeV)	$g_\alpha^2/4\pi$	Λ_α(GeV)
π	138.03	14.7	1.3
η	548.8	4	1.5
ρ	769	0.86	1.95
ω	782.6	25	1.35
δ	983	1.3	2.0
σ [b]	550	8.8	2.0
	(710–720)	(17.194)	(2.0)

The Lagrangian in Bonn A potential

Lagrangians for meson-nucleon couplings are

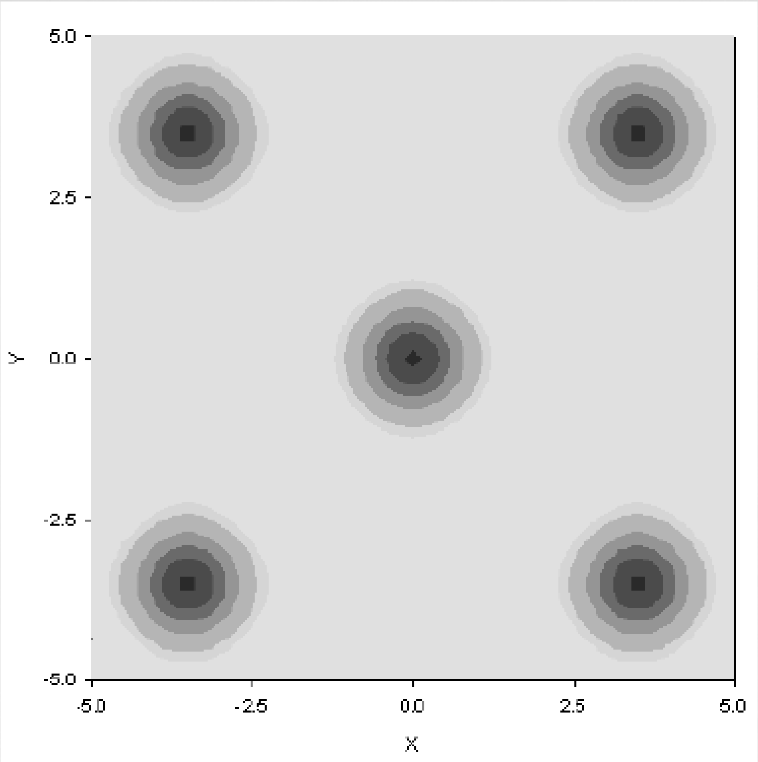
$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi}i\gamma^5\psi\varphi^{(ps)}$$

$$\mathcal{L}_{pv} = -\frac{f_{ps}}{m_{ps}}\bar{\psi}\gamma^5\gamma^\mu\psi\partial_\mu\varphi^{(ps)}$$

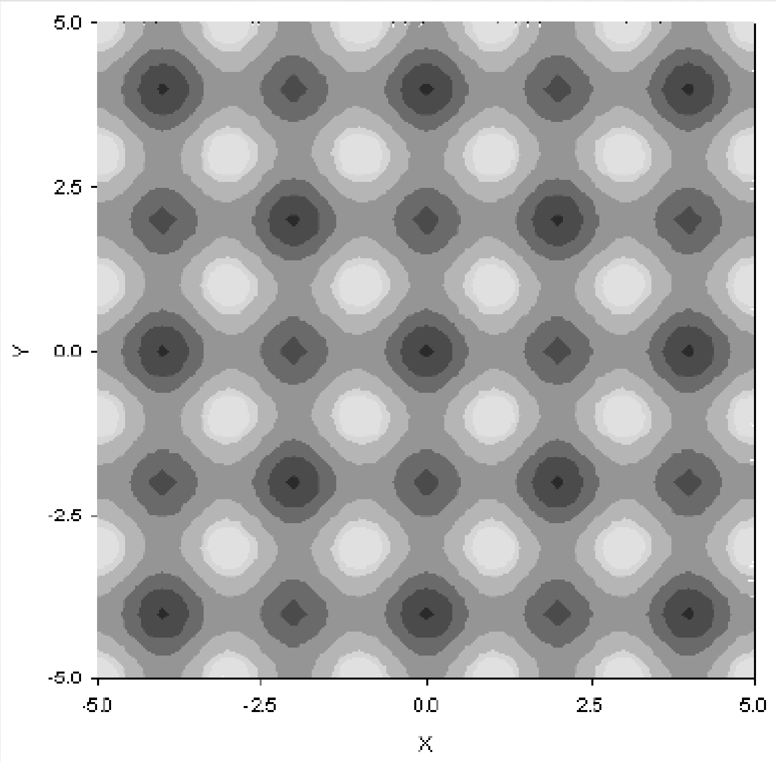
$$\mathcal{L}_s = +g_s\bar{\psi}\psi\varphi^{(s)}$$

$$\mathcal{L}_v = -g_v\bar{\psi}\gamma^\mu\psi\varphi_\mu^{(v)} - \frac{f_v}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_\mu\varphi_\nu^{(v)} - \partial_\nu\varphi_\mu^{(v)})$$

H. K. Lee, B.-Y. Park and M. Rho, Phys. Rev. C **83**, 025206 (2011)
[Erratum-ibid. C 84, 059902 (2011)]



$$n = n_{1/2}$$



$$\frac{f_\pi^*}{f_\pi} \approx 1 - D_I\left(\frac{n}{n_0}\right)$$

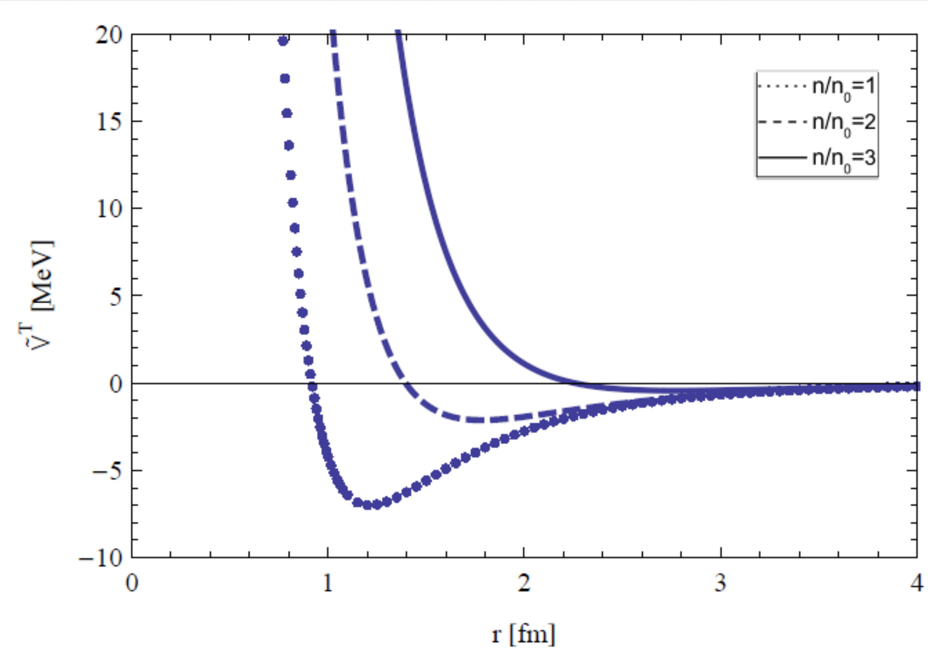
$$n < n_{1/2}$$

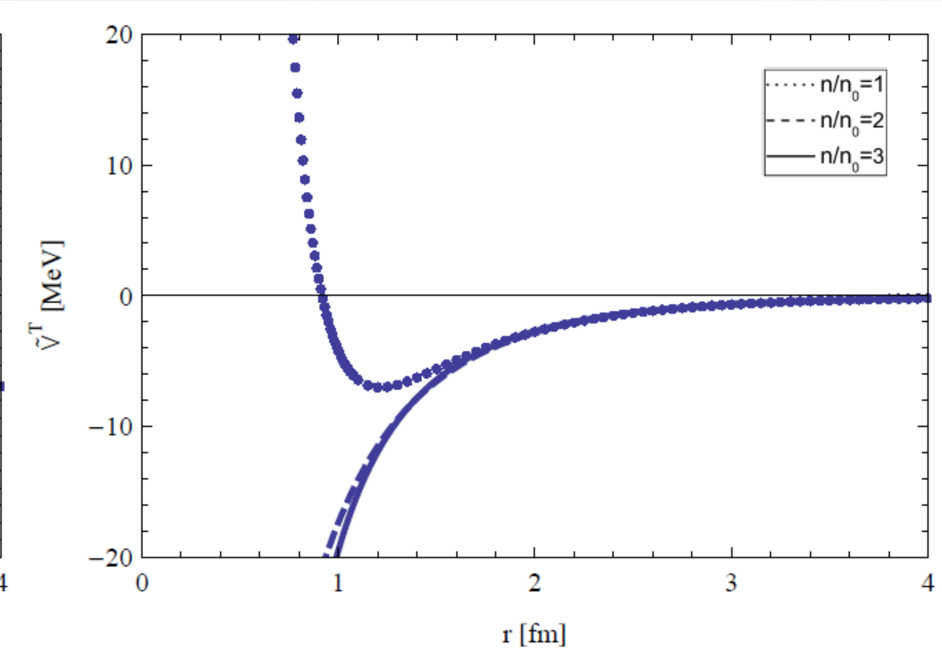
$$\frac{f_\pi^*}{f_\pi} \approx c \neq 0$$

$$n_{1/2} < n$$

Without topology change

With topology change





, where $D_I = D_{II} = 0.15$ was considered. So, we expect that the tensor force by exchanging ρ meson will be suppressed and only pion tensor force will remain.

$V^T(r) = V^T_\rho(r) + V^T_\pi(r)$

$$V_M^T(r) \quad = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

where $M = \pi, \rho,\ S_{\rho(\pi)} = +1(-1)$.

$$R \equiv \frac{f^*_{N\rho}}{f_{N\rho}} \approx \frac{g^*_{\rho NN}}{g_{\rho NN}} \frac{m^*_\rho}{m_\rho} \frac{m_N}{m^*_N}.$$

Hatsuda and Kunihiro yielded the in-medium Gell-Mann-Oakes-Renner relation,

$$m^*_\pi(n)/m_\pi \approx (f^t_\pi(n)/f_\pi)^{-1} (\langle \bar{q}q \rangle^*(n)/\langle \bar{q}q \rangle)^{1/2}$$

Using the experimental information available at the nuclear matter density, $(f^t_\pi(n_0)/f_\pi)^2 \simeq 0.64$ and $\langle \bar{q}q \rangle(n_0)/\langle \bar{q}q \rangle \simeq 0.63$, we get $m_\pi{}^*/m_\pi \simeq 1$.

Tensor Force and Symmetry Energy

$$E(n,\delta) = E_0(n) + E_{sym}(n)\delta^2 + \cdots$$

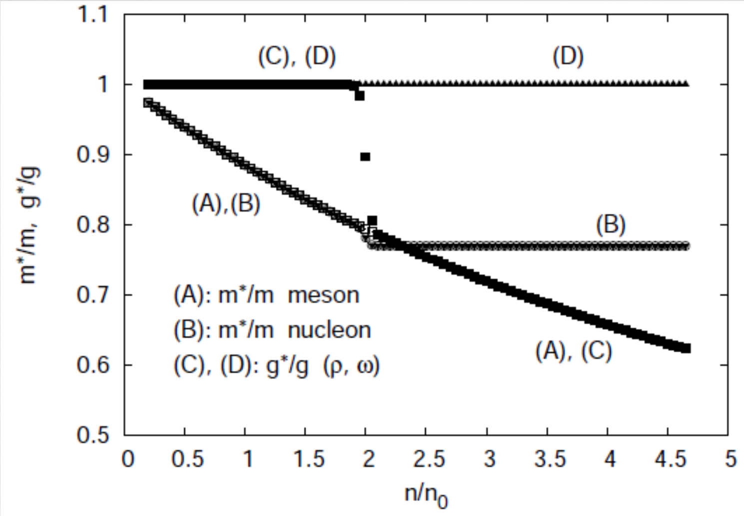
$$\delta = (N-Z)/(N+Z)$$

where E is the energy per baryon of the system,

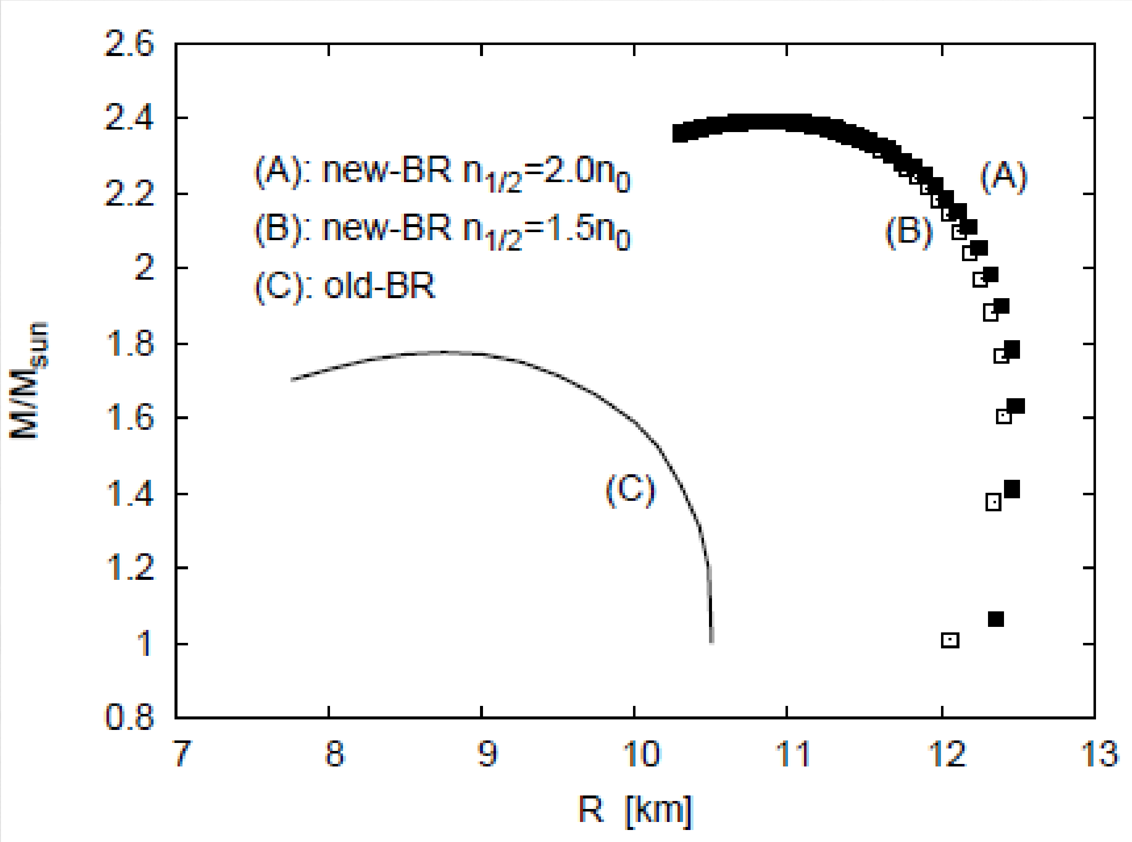
$$E_{sym} \sim \langle V_{sym} \rangle \approx \frac{12}{\bar{E}} \langle V_T^2(r) \rangle$$

where $\bar{E} \approx 200$ MeV is the average energy typical of the tensor force excitation and V_T is the radial part of the net tensor force.

G. E. Brown and R. Machleidt, Phys. Rev. C 50, 1731 (1994)

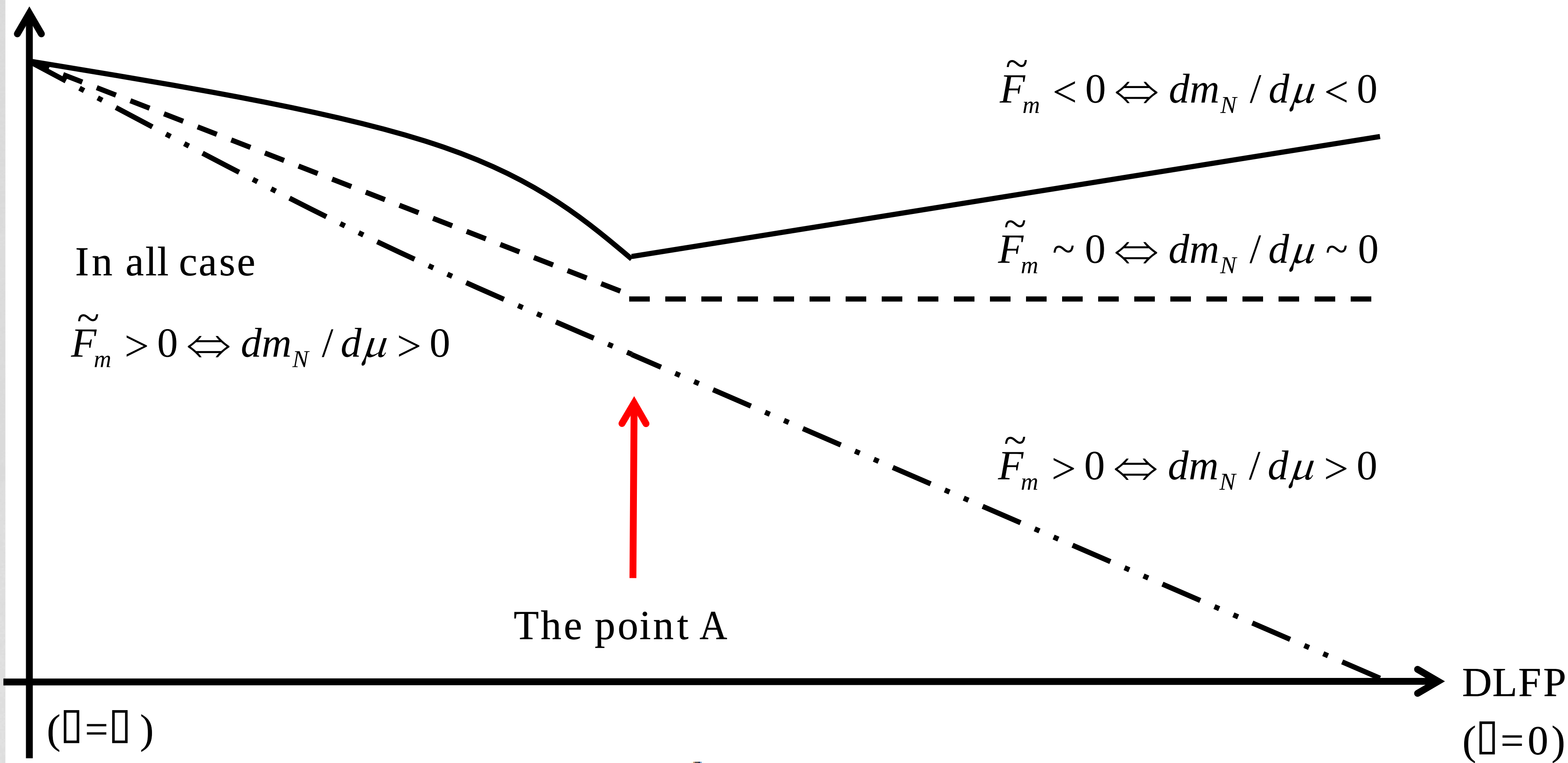


H. Dong, T. T. S. Kuo, H. K. Lee, R. Machleidt and M. Rho, Phys. Rev. C 87, no. 5, 054332 (2013)

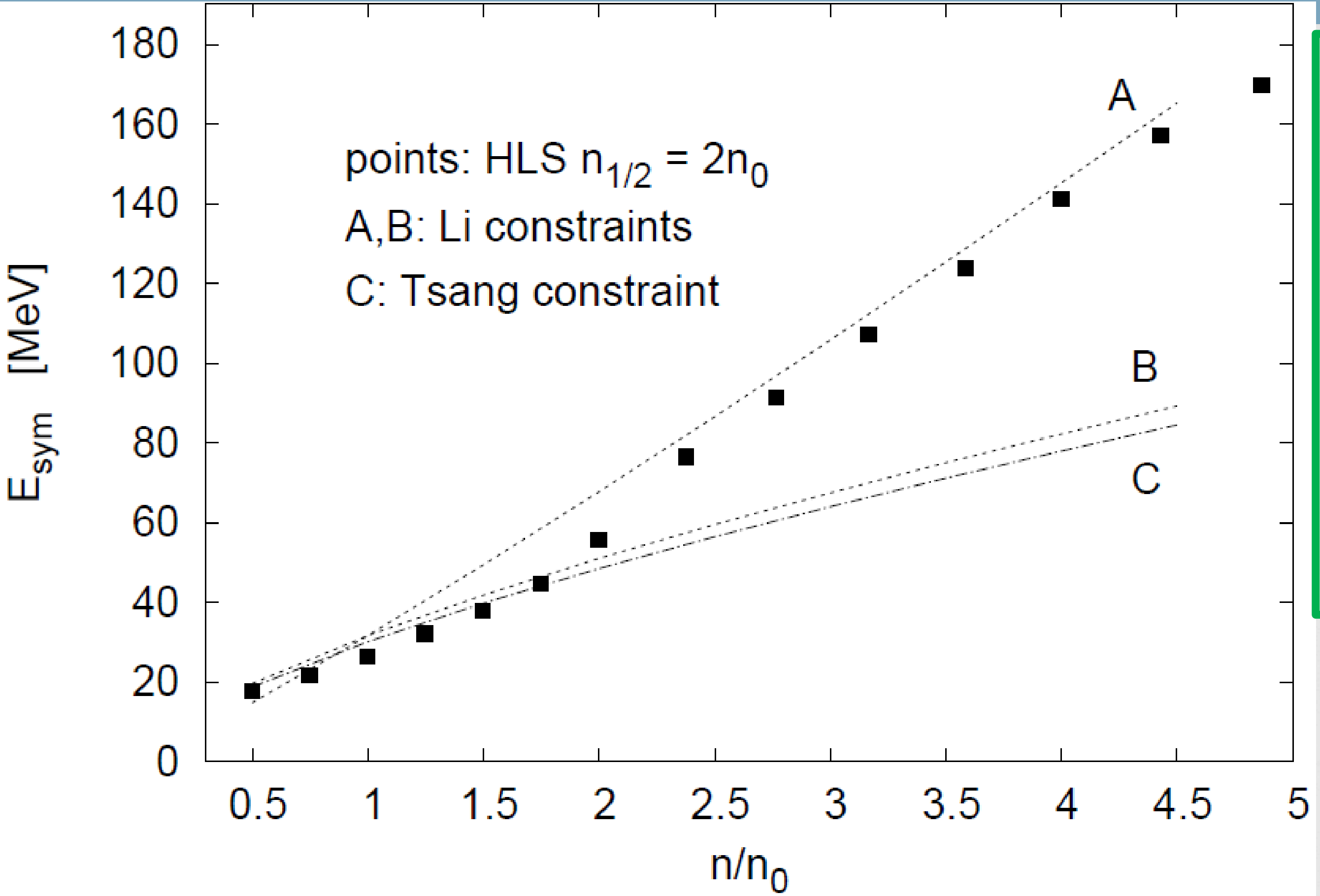


It shows the interplay between the nucleon mass and the ω-NN coupling

$$\mu \frac{dm_N}{d\mu} = \frac{3m_N}{16\pi^2}\tilde{\mathcal{F}}_m \,,$$



$$\tilde{\mathcal{F}}_m \;=\; \frac{g_A^2}{F_\pi^2}\left(\mu^2 - m_N^2\right) - \frac{3}{2}\left(1 - g_{V\rho}\right)^2 g_\rho^2 - \frac{1}{2}\left(g_{V\omega} - 1\right)^2 g_\omega^2$$



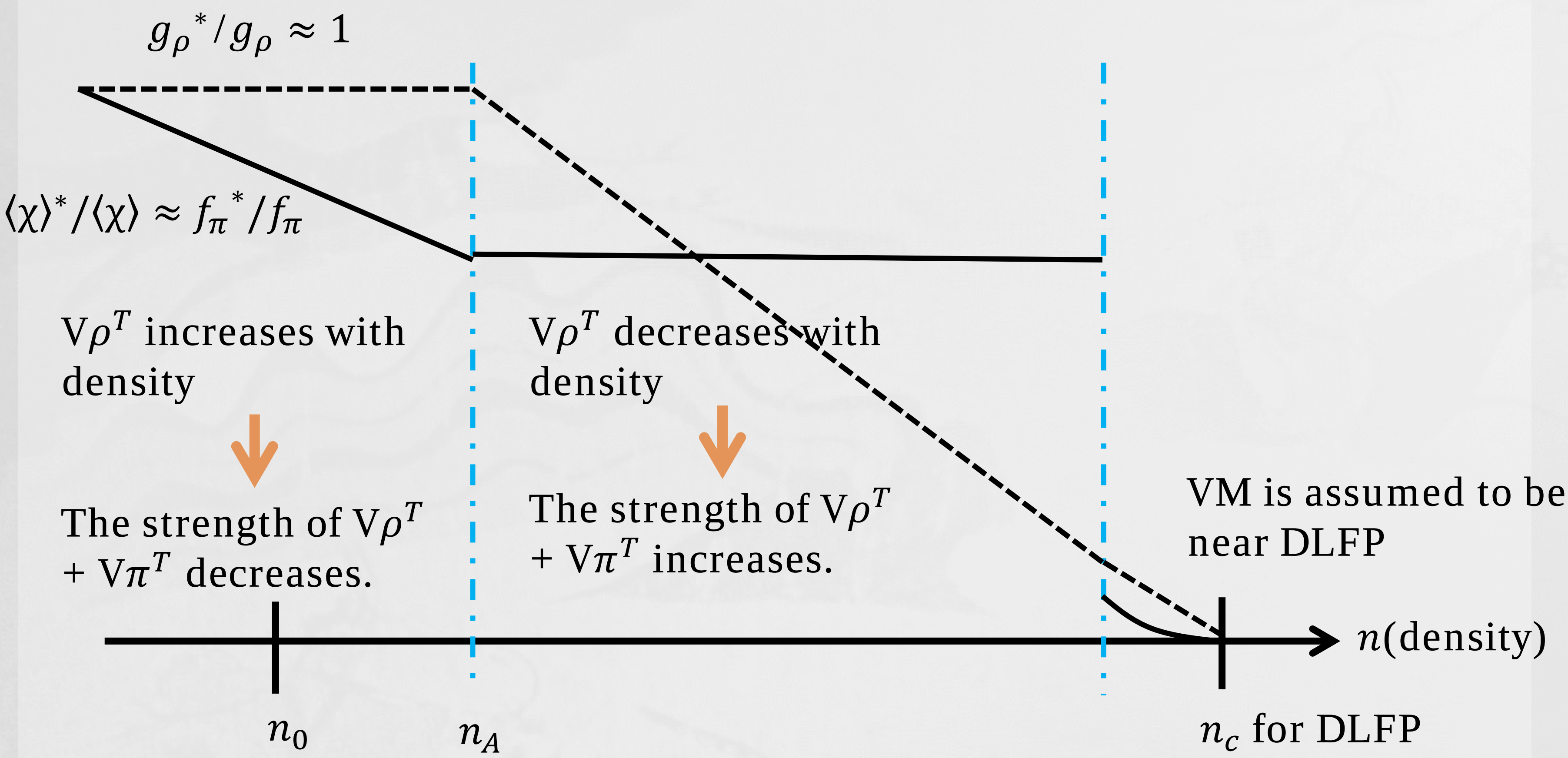
Li:
$E_{sym}(n) \approx 31.6(n/n_0)^{\gamma}$;
$\gamma = 0.69(\mathrm{B}) - 1.1(\mathrm{A})$

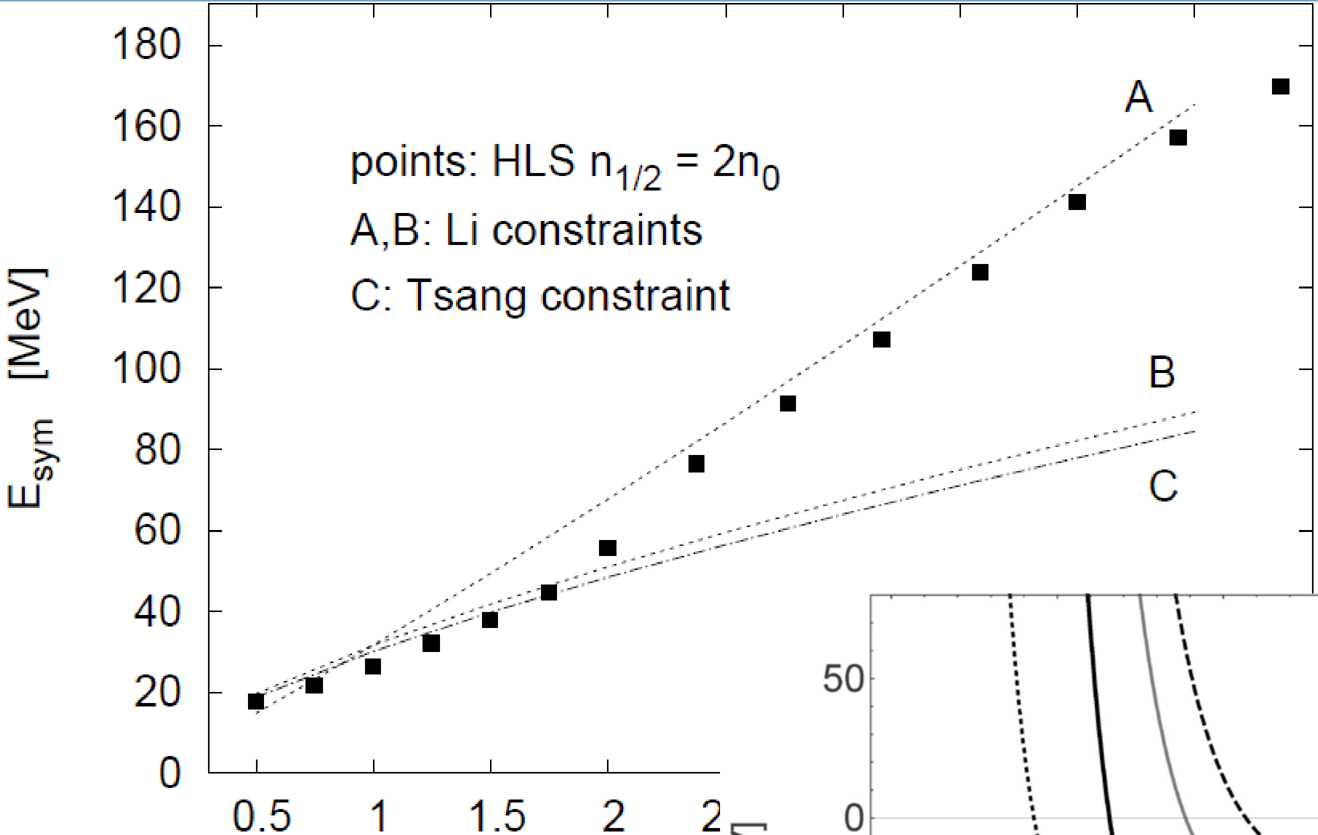
Tsang:
$E_{sym}(n) = \frac{C_{s,k}}{2}\left(\frac{n}{n_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{n}{n_0}\right)^{\gamma_i}$;
$C_{s,k} = 25$ MeV, $C_{s,p} = 35.2$ MeV
and $\gamma_i \approx 0.7$(C).

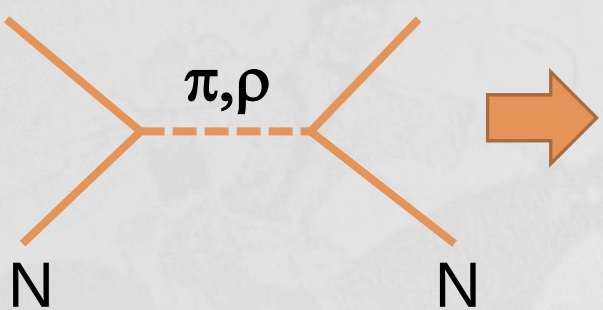
E_{sym}/MeV	L/MeV	
27	57.3	*bs*HLS
31.6	65.4-104.2	Li
30.1	62.0	Tsang
28-32	40-60	Lattimer

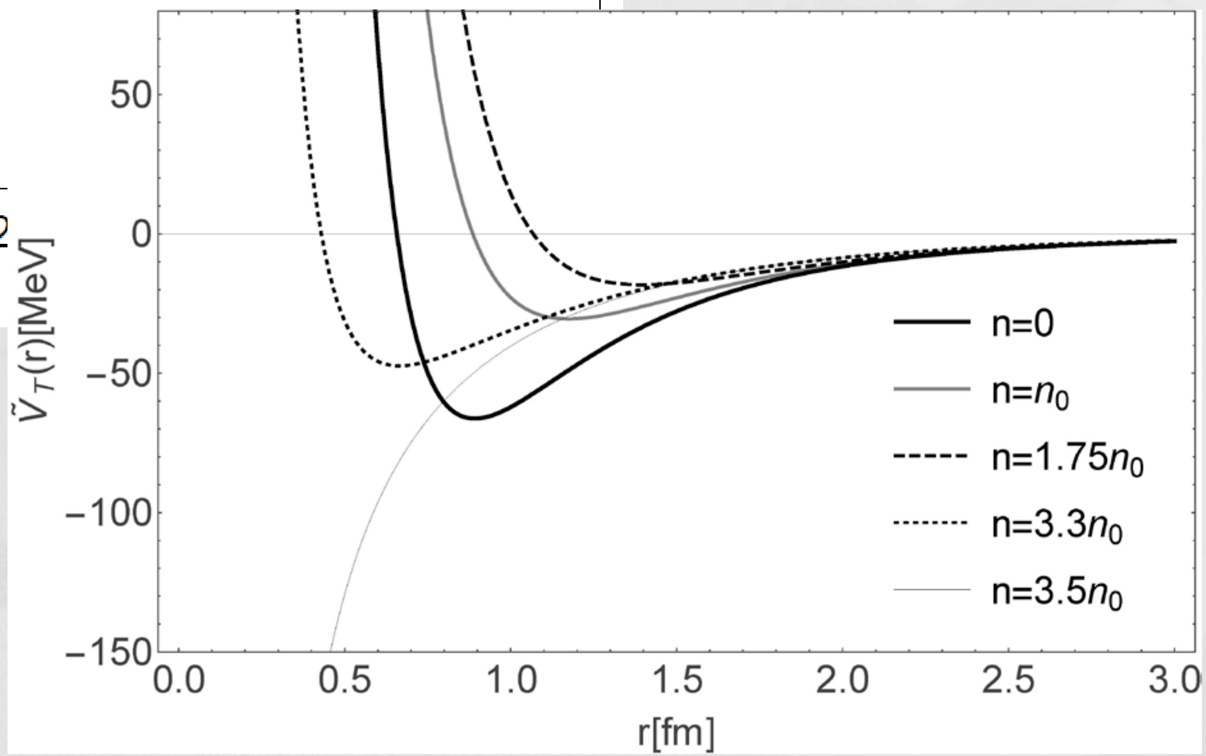
[Li]B. A. Li and L. W. Chen, Phys. Rev. C **72**, 064611 (2005).
[Tsang] M. B. Tsang, Y. Zhang, P. Danielewicz, M. Famiano, Z. Li, W. G. Lynch and A. W. Steiner, Phys. Rev. Lett. **102**, 122701 (2009) [Int. J. Mod. Phys. E **19**, 1631 (2010)] .
[Lattimer] J. M. Lattimer and Y. Lim, Interaction, Astrophys. J. **771**, 51 (2013).

Prediction for the symmetry energy




points: HLS $n_{1/2} = 2n_0$
A,B: Li constraints
C: Tsang constraint
A
B
C
E_{sym} [MeV]
180
160
140
120
100
80
60
40
20
0
0.5
1
1.5
2


π,ρ
N
N


$\tilde{V}_T(r)$[MeV]
r[fm]
n=0
n=n_0
n=1.75n_0
n=3.3n_0
n=3.5n_0
50
0
−50
−100
−150
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Prediction for Sound velocity

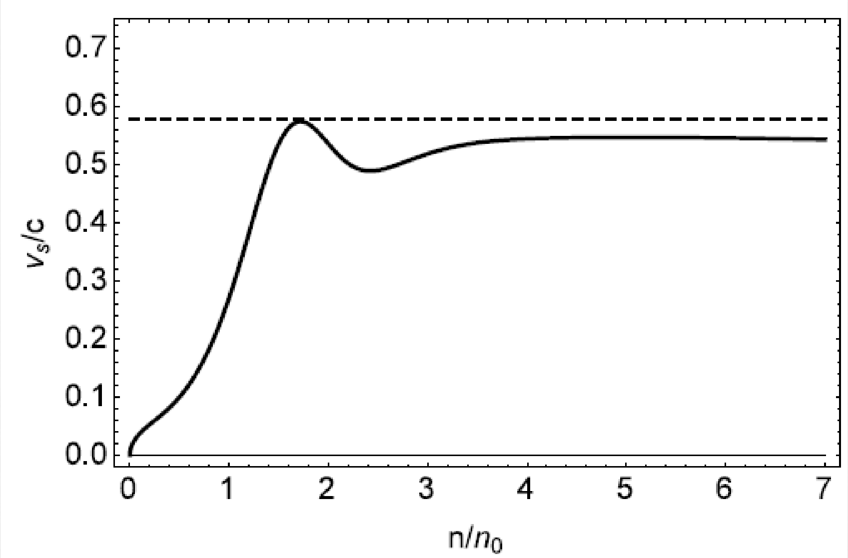
$\langle\theta^\mu_\mu\rangle = \epsilon(n) - 3P(n)$

The energy-momentum tensor and the sound velocity are related by

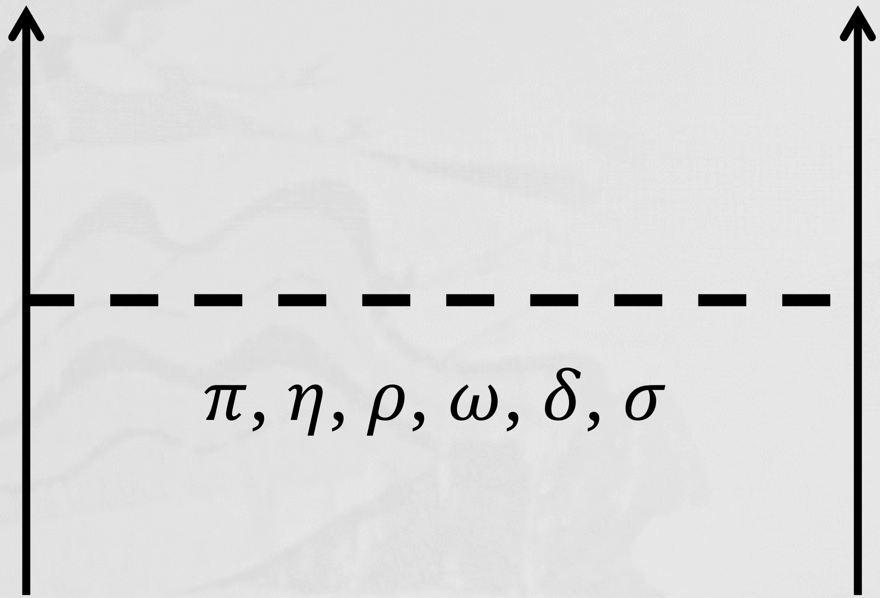
$$\frac{\partial}{\partial n}\langle\theta^\nu_\nu\rangle = \frac{\partial\,\epsilon(n)}{\partial n}\left(1 - 3\frac{v_s^2}{c^2}\right),$$

where we used $\frac{v_s^2}{c^2} = \frac{\partial\,P(n)}{\partial n}/\frac{\partial\,\epsilon(n)}{\partial n}$ for the sound velocity v_s.

$\langle\theta^\mu_\mu\rangle = \langle 4V(\chi) - \chi\partial V(\chi)/\partial\chi\rangle \propto -m_\chi^{*\,2}\langle\chi\rangle^{*\,2}.$

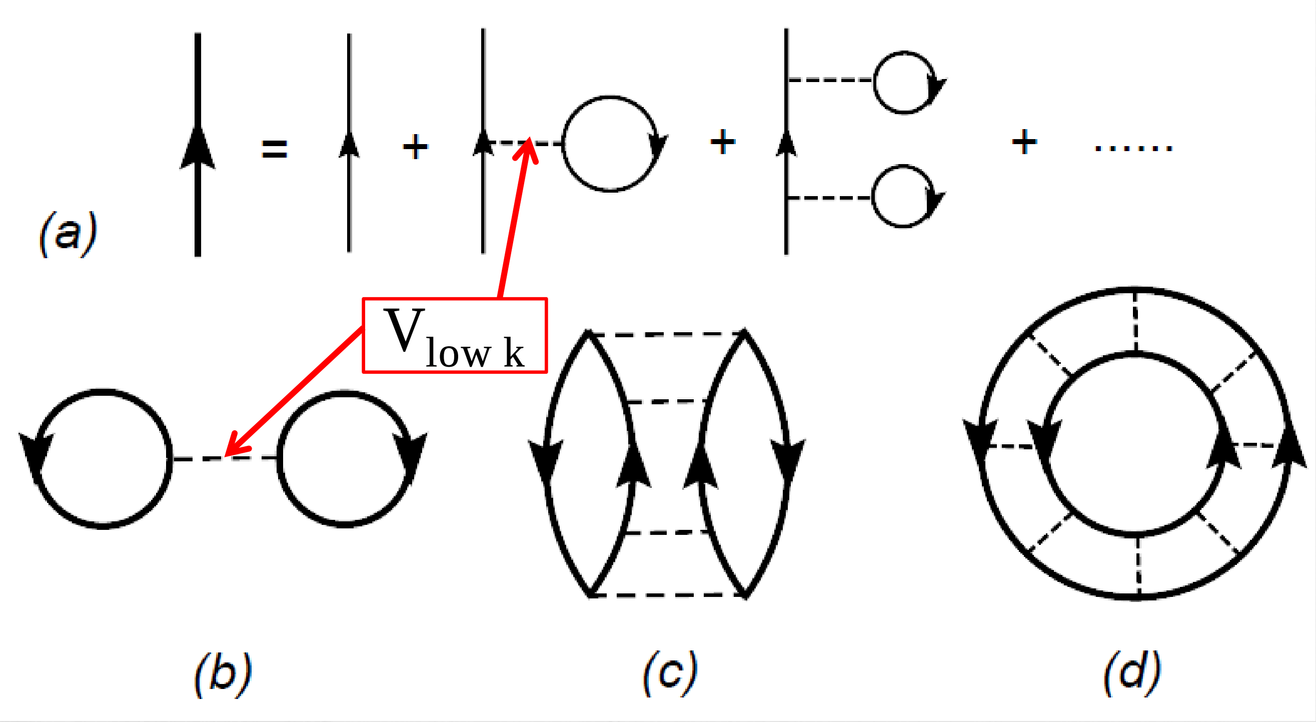


- The Bonn A potential is a phenomenological potential given by one boson exchange expressed as



- When we go to the nuclear matter, the density dependence of the parameters obtained in bsHLS for the mass and coupling constants are used for π, ρ, ω, σ as $\frac{m^*}{m} = \frac{\langle \chi \rangle}{f_{0\sigma}}$ and $\frac{g^*}{g}$.

- We obtain the ground state energy of the symmetric nuclear matter and pure neutron matter by calculating the single particle energy for the diagram (a) and the pphh ring diagrams for the diagram (b), (c) and (d) summed to all orders within a model space of the cutoff Λ.



- The effective Lagrangian for the low energy region, which accounts for the explicit scale symmetry breaking of QCD, can be constructed by implementing a dilaton χ which transforms as $\chi \rightarrow \lambda\, \chi$ under the scale transformation.
- The terms for the explicit scale symmetry breaking are given by

$$-V(\chi) + \frac{{f_{0\pi}}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{tr}[MU^+ + h.c.]$$

in the effective Lagrangian, where $M = 2B^0\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$. The second term in the above equation is given to have the same scale dimension as '$m_q\bar{q}q$'. If we take Coleman-Weinberg potential for $V(\chi)$ as $V(\chi) = \frac{B}{4}\chi^4 \mathrm{tr}[\ln\left(\frac{\chi}{f_{0\sigma}}\right)^4 - 1]$, we get

$$\left(\theta^{\mu}{}_{\mu}\right)_{eff} = B\chi^4 + \frac{{f_{0\pi}}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{tr}[MU^+ + h.c.].$$

- At the matching scale $-q^2 = Q^2 \approx {\Lambda_M}^2$, the effective Lagrangian is matched with QCD by

$$\left\langle \left(\theta^{\mu}{}_{\mu}\right)_{QCD} \right\rangle = \left\langle \left(\theta^{\mu}{}_{\mu}\right)_{eff} \right\rangle,$$

so $\langle \chi \rangle$ carries the information of the density dependence by relating it to $\langle \bar{q}q \rangle$ and $\langle G^2 \rangle$. We call this density dependence Intrinsic Density Dependence(IDD).

- We would like to construct the scale-invariant effective model(χPT) with a scalar field(dilaton), where ρ and ω are given as a gauge boson of the hidden local symmetry and the baryon also included. We call this "bsHLS". We show that the parameters in the effective model are related to $\langle \bar{q}q \rangle$ and $\langle G^2 \rangle$ by $\langle \chi \rangle$.

We are interested in the terms,

$$\begin{aligned}\mathcal{L}_{bs\mathrm{HLS}} &= \frac{1}{2}\left(\frac{\chi}{f_{0\sigma}}\right)^2 \partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{2}\partial^\mu \chi \partial_\mu \chi + \bar{N} i \gamma^\mu \partial_\mu N - \sum_{V=\rho,\omega} \frac{1}{2}\mathrm{tr}\left[V_{\mu\nu}V^{\mu\nu}\right] \\ &+ \sum_{V=\rho,\omega} \frac{1}{2} m_V^2 \left(\frac{\chi}{f_{0\sigma}}\right)^2 V^\mu V_\mu - \frac{f_{0\pi}^2}{2} m_\pi^2 \left(\frac{\chi}{f_{0\sigma}}\right)^3 \frac{\pi^a \pi_a}{f_{0\pi}^2} + V(\chi) - m_N \frac{\chi}{f_{0\sigma}} \bar{N} N \\ &- \sum_{V=\rho,\omega} g_V \left(g_{VN} - 1\right) \bar{N} \gamma_\mu V^\mu N + g_A \bar{N} \gamma^\mu \gamma_5 \frac{\partial_\mu \pi}{f_{0\pi}} N + \cdots,\end{aligned}$$

where one boson exchange NN interactions are shown in the leading order of scale-chiral counting. Here, $V_\mu = \rho_\mu^a \frac{\tau^a}{2}$ or $\frac{\omega_\mu}{2}$, $\pi = \pi^a \frac{\tau^a}{2}$, $m_V^2 = f_{0\sigma_V}^2 g_V^2$ and

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i g_V \left[V_\mu, V_\nu\right].$$

$$\boxed{(m_\pi)^2 = 2B^0 m,\ m_u = m_d = m}$$

If we define σ as

$$\chi = f_{0\sigma} \exp\left(\frac{\sigma}{f_{0\sigma}}\right)$$

and take $\sigma \to \langle\sigma\rangle - \sigma'$, $\mathcal{L}_{bs\mathrm{HLS}}$ is rewritten as

$$\begin{aligned}
\mathcal{L}_{bs\mathrm{HLS}} \;=\;& \frac{1}{2}\partial_\mu \pi^{*\,a}\partial^\mu \pi^{*\,a} + \frac{1}{2}\partial^\mu \sigma^* \partial_\mu \sigma^* + \bar{N} i\gamma^\mu \partial_\mu N - \sum_{V=\rho,\omega} \frac{1}{2}\mathrm{tr}\left[V_{\mu\nu}V^{\mu\nu}\right] \\
&+ \sum_{V=\rho,\omega} \frac{1}{2} m_V^{*\,2} V^\mu V_\mu - \frac{1}{2} m_\pi^{*\,2}\left(\pi^{*\,a}\right)^2 + \frac{1}{2} m_\sigma^{*\,2} \sigma^{*\,2} \\
&- m_N^* \bar{N} N + g_\sigma \sigma^* \bar{N} N - \sum_{V=\rho,\omega} g_{VNN} \bar{N}\gamma_\mu V^\mu N + g_A \bar{N}\gamma^\mu \gamma_5 \frac{\partial_\mu \pi^*}{f_\pi^*} N + \cdots\,,
\end{aligned}$$

where π^* and σ^* are defined as $\pi^* = \frac{\langle\chi\rangle}{f_{0\sigma}}\pi$ and $\sigma^* = \frac{\langle\chi\rangle}{f_{0\sigma}}\sigma'$, $\langle\chi\rangle = f_{0\sigma}\exp\left(\frac{\langle\sigma\rangle}{f_{0\sigma}}\right)$ and

$$\begin{aligned}
m_V^* &= g_V f_{0\sigma V} \frac{\langle\chi\rangle}{f_{0\sigma}},\; m_\pi^* = m_\pi \left(\frac{\langle\chi\rangle}{f_{0\sigma}}\right)^{1/2},\; m_N^* = m_N \frac{\langle\chi\rangle}{f_{0\sigma}},\; f_\pi^* = f_\pi \frac{\langle\chi\rangle}{f_{0\sigma}} \\
m_\sigma^{*\,2} &= -\left.\frac{\partial^2}{\partial\sigma^{*\,2}} V\left(\langle\chi\rangle - \sigma^*\right)\right|_{\sigma^*=0} \approx m_\sigma^2 \left(\frac{\langle\chi\rangle}{f_{0\sigma}}\right)^2, \\
g_\sigma &= \frac{m_N}{f_{0\sigma}},\; g_{\rho NN} = g_\rho\left(g_{\rho N} - 1\right) \text{ and } g_{\omega NN} = g_\omega\left(g_{\omega N} - 1\right).
\end{aligned}$$

Application to Dense nuclear matter

- Now, we get the three free parameters in the model, which has a

density dependence. They are $\langle\chi\rangle$, g_ρ and g_ω. The all other parameters are related to them by

$$\frac{m^*{}_N}{m_N} \approx \frac{m^*{}_\sigma}{m_\sigma} \approx \frac{f^*{}_\pi}{f_{0\pi}} \approx \frac{\langle\chi\rangle}{f_{0\sigma}} \;\&\; \frac{m^*{}_V}{m_V} \propto g_V \frac{\langle\chi\rangle}{f_{0\sigma}}$$

$$g_{VNN} \propto g_V,$$

where $V = \rho$ and ω. Here, please note that g_ρ also has a density dependence by matching the current correlators of HLS with the current correlators of QCD at the matching scale Λ_M.

- $\langle\chi\rangle$ is defined as the classical solution for the equation of the motion of χ given by

$$\frac{\partial}{\partial\chi}\mathcal{L}(\chi)|_{\chi=\langle\chi\rangle} = 0.$$

If we go into the nuclear matter, the value of $\langle\chi\rangle$ will be changed with depending on $\langle N^+N\rangle$ because χ is coupled with a nucleon by $\sim\chi\bar{N}N$.

The notation of the particles

Coupling	Bosons (Strength of Coupling)		Characteristics of Predicted Forces			
	$I = 0$ $[1]$	$I = 1$ $[\tau_1 \cdot \tau_2]$	Central $[1]$	Spin-Spin $[\sigma_1 \cdot \sigma_2]$	Tensor $[S_{12}]$	Spin-Orbit $[L \cdot S]$
ps	η (weak)	π (strong)	—	weak, coherent with v, t	strong	—
s	σ (strong)	δ (weak)	strong, attractive	—	—	coherent with v
v	ω (strong)	ρ (weak)	strong, repulsive	weak coherent with ps	opposite to ps	strong, coherent with s
t	ω (weak)	ρ (strong)	—	weak, coherent with ps	opposite to ps	—

I denotes the isospin of a boson. The characteristics quoted refer to $I = 0$ bosons (no isospin dependence). The isovector ($I = 1$) boson contributions, carrying a factor $\tau_1 \cdot \tau_2$, provide the isospin-dependent forces.

Pion tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^{*\,2}}{4\pi} \tau_1 \tau_2 S_{12} \mathcal{I}(m_M^* r)$$

$$\mathcal{I}(m_M^* r) \equiv m_M^* \left(\left[\frac{1}{(m_M^* r)^3} + \frac{1}{(m_M^* r)^2} + \frac{1}{3 m_M^* r} \right] e^{-m_M^* r} \right)$$

where $M = \pi, \rho$, $S_{\rho(\pi)} = +1(-1)$ and

$$S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

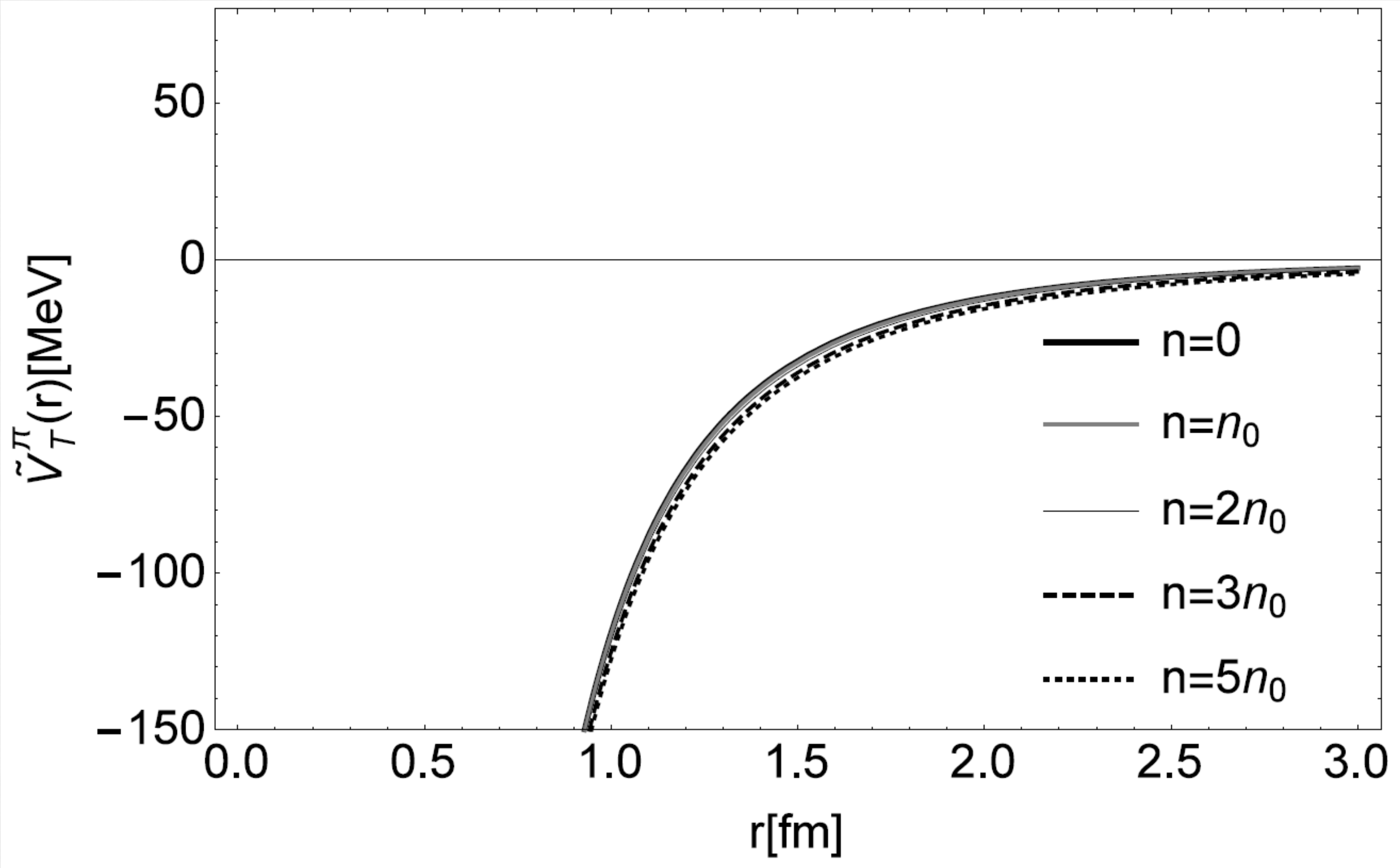
$$R_\pi \approx \frac{g_{\pi NN}^*}{g_{\pi NN}} \frac{m_N}{m_N^*} \frac{m_\pi^*}{m_\pi}$$

$$\approx \begin{cases} \Phi_I \times \Phi_I^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-I} \\ \kappa \times \kappa^{-1} \left(\frac{m_\pi^*}{m_\pi} \right) & \text{for R-II} \end{cases}$$

$$f_\pi^{*\,2} m_\pi^{*\,2} = m_q \langle \bar{q} q \rangle + \sum_{n>1} c_n \langle (\bar{q} q)^n \rangle$$

$$\frac{m_N^*}{m_N} \approx \frac{g_V}{g_V^*} \frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \left(\frac{m_\pi^*}{m_\pi} \right)^2 \approx \frac{f_\pi^*}{f_\pi} \approx \frac{\langle \chi \rangle^*}{f_\sigma}$$

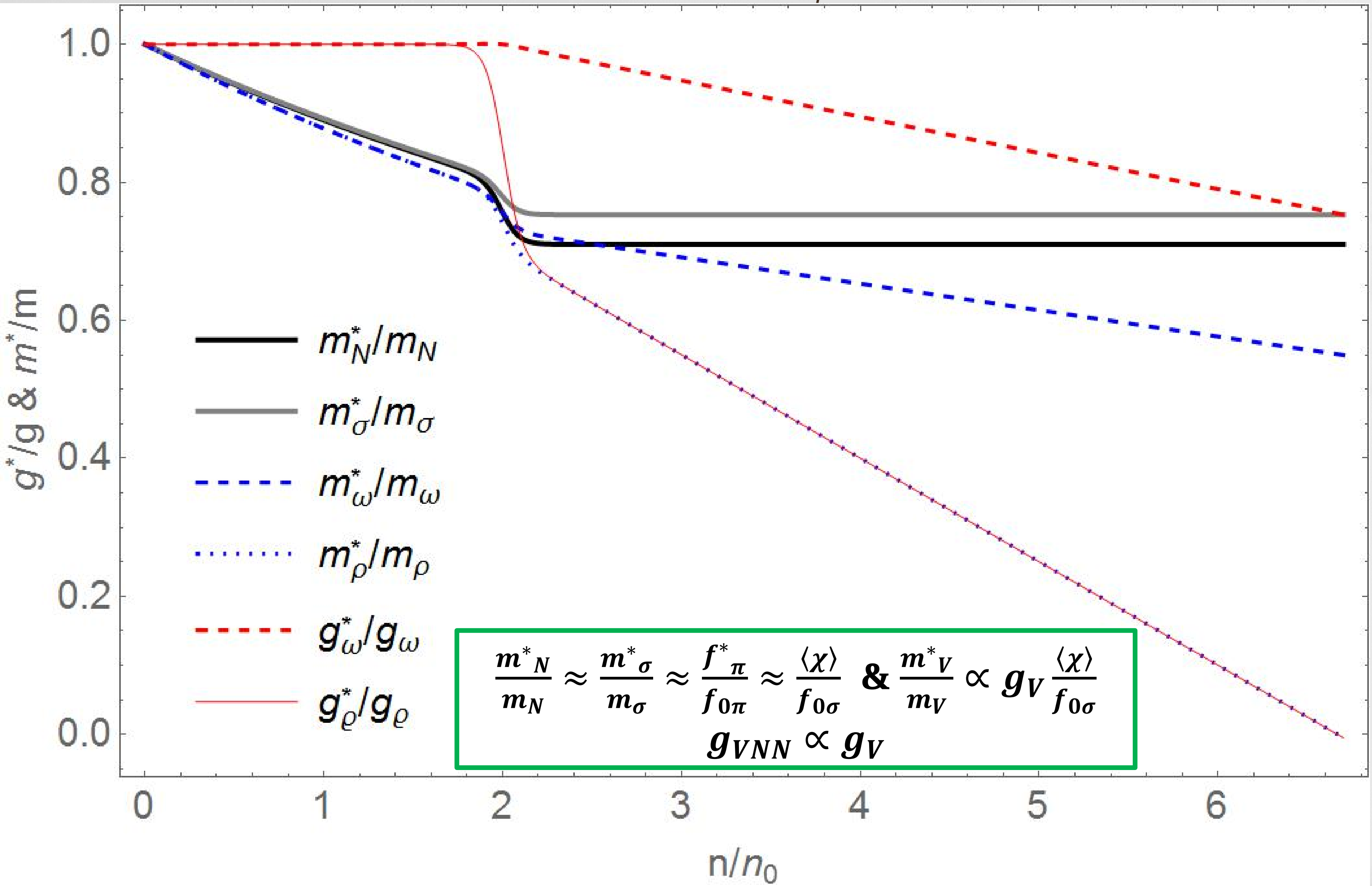
$$\frac{m_\pi^*}{m_\pi} = \left(\frac{1}{1+0.13*n/n_0}\right)^{1/2} \frac{1}{1+\exp\left(\frac{n-n_{1/2}}{0.05n_0}\right)} + \left(1-0.15*\frac{n}{n_0}\right)\frac{1}{1+\exp\left(-\frac{n-n_{1/2}}{0.05n_0}\right)}$$

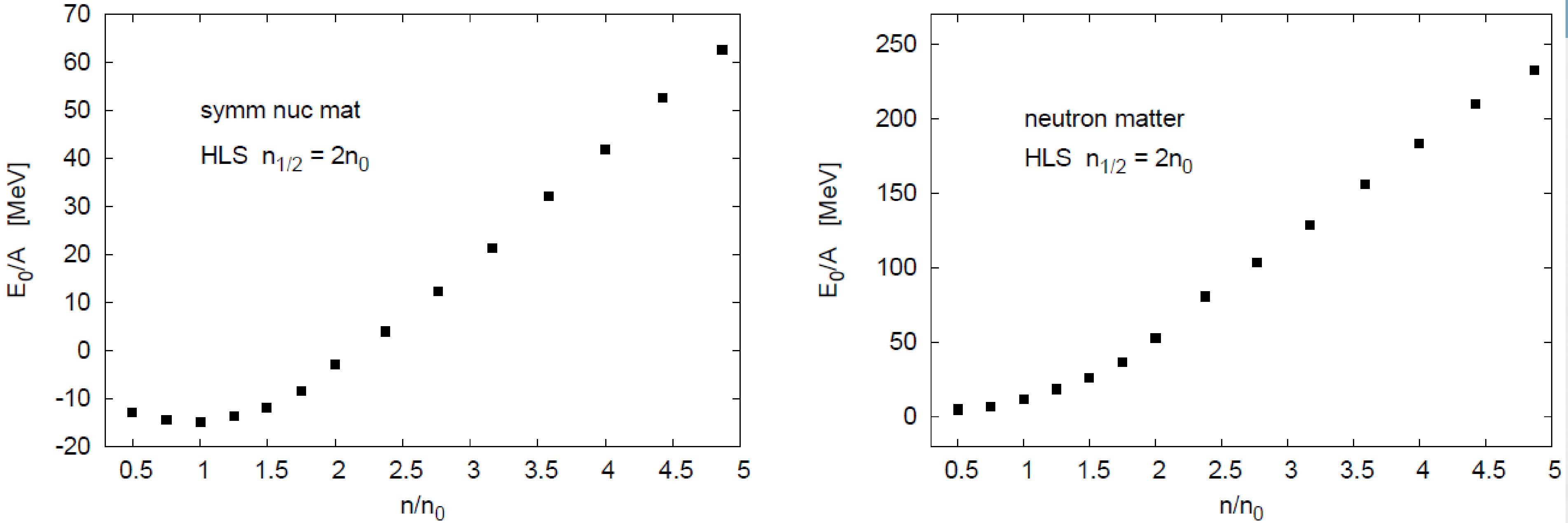


Application to Dense nuclear matter

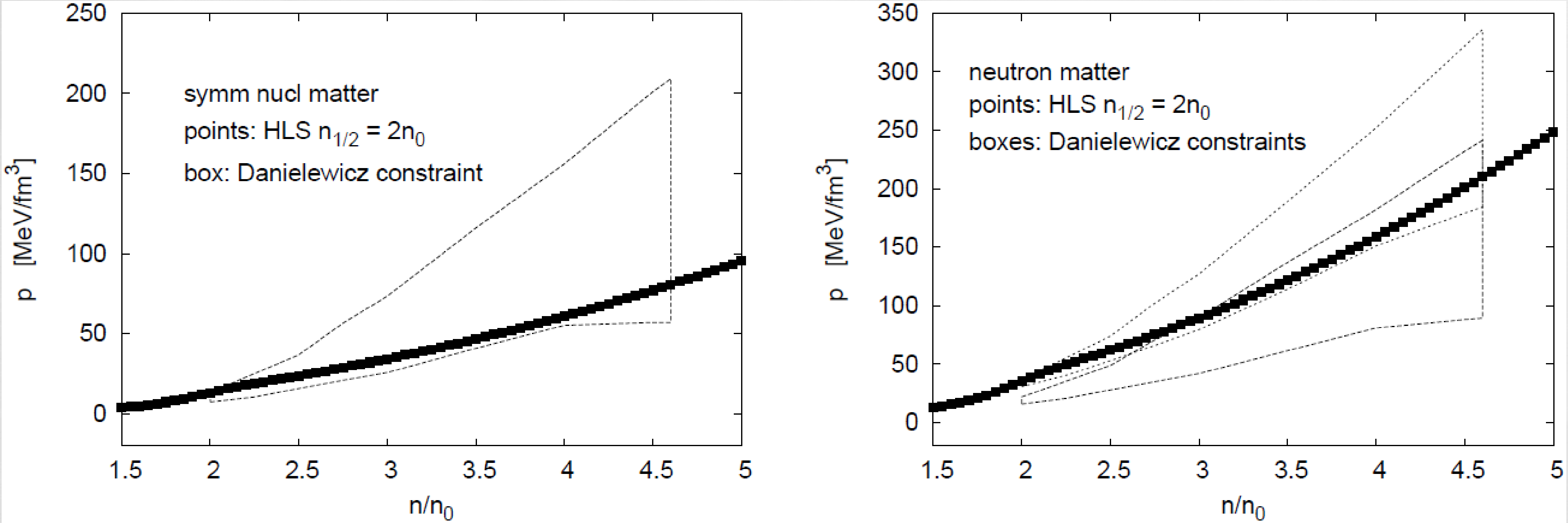
Result

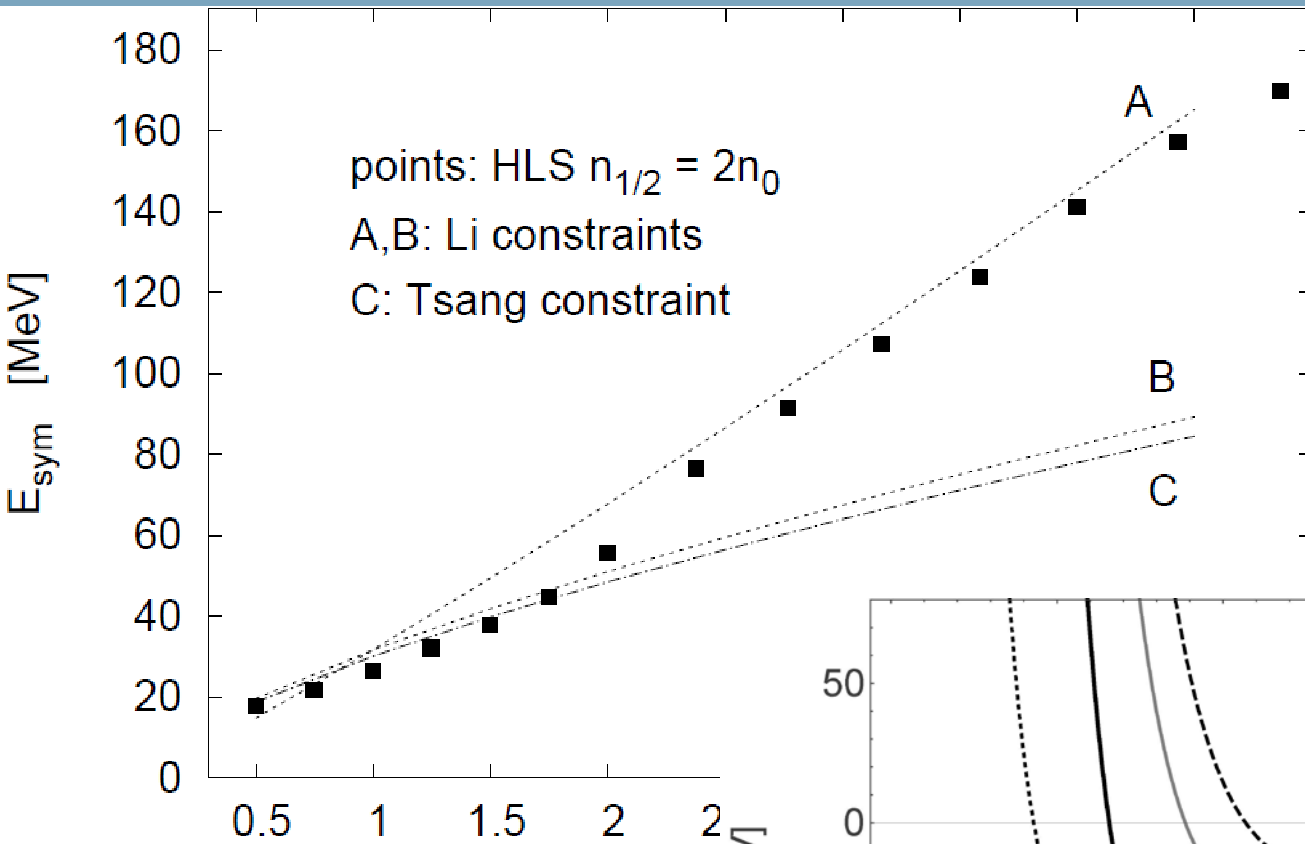
- We obtain the best scaling of the parameters which give the reasonable EoS for the dense nuclear matter and satisfy VM property of $m^*{}_\rho \sim g^*{}_\rho \sim \langle \bar{q}q \rangle \rightarrow 0$ at the critical density.

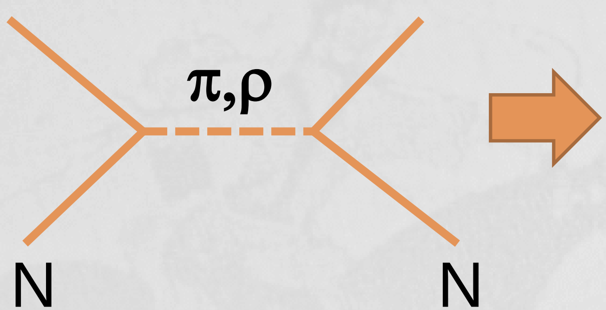


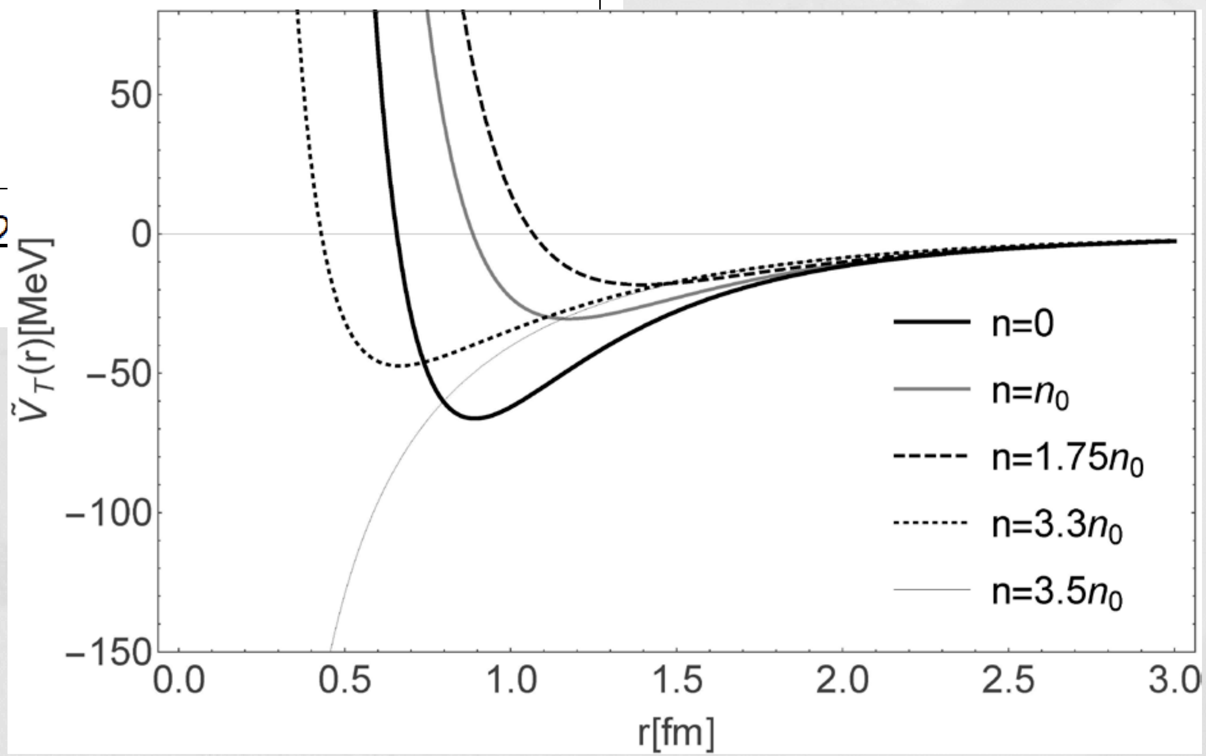


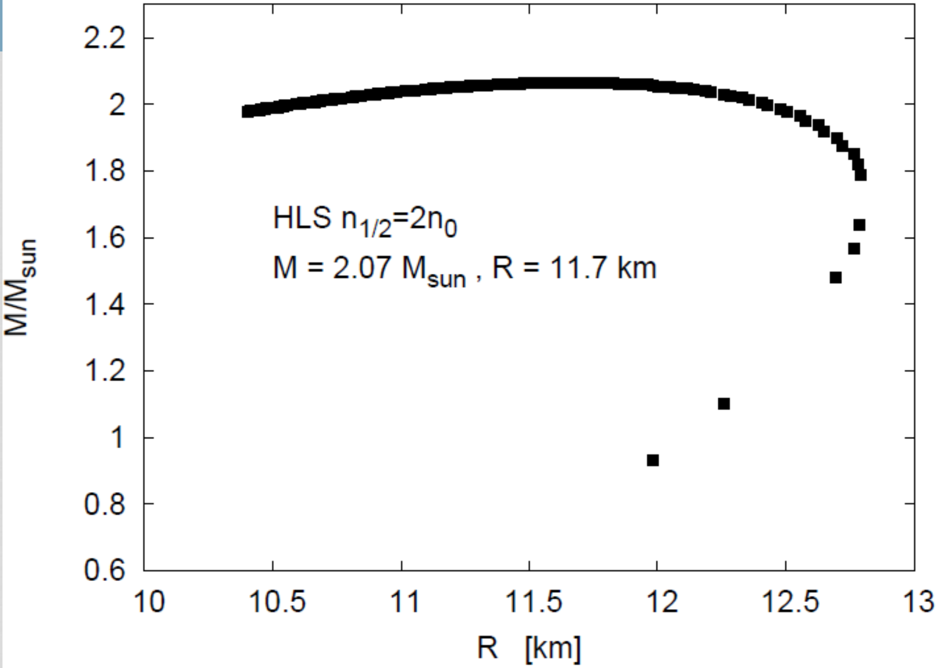
The scaling gives, for symmetric nuclear matter, the saturation energy E_0/A=-15.1 MeV, saturation density $n_{sat} \approx 0.16\ \mathrm{fm}^{-3}$ and compression modulus K = 183.2 MeV.

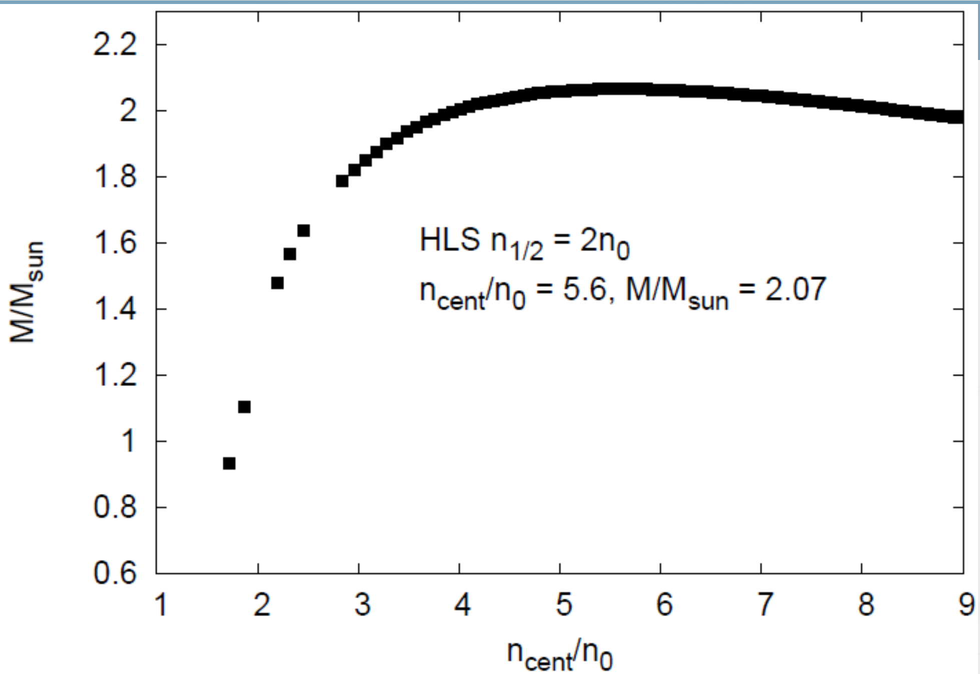


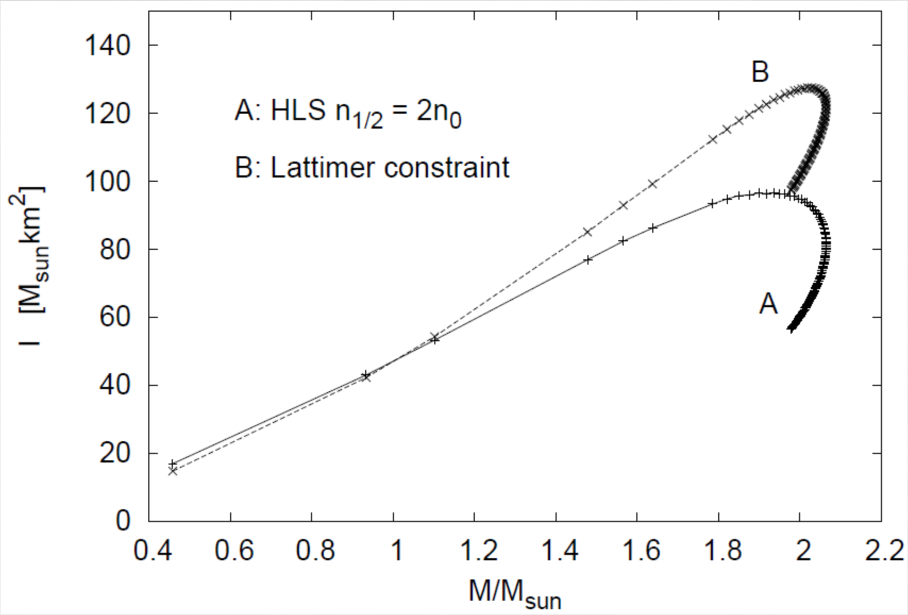

points: HLS $n_{1/2} = 2n_0$
A,B: Li constraints
C: Tsang constraint
A
B
C
E_{sym} [MeV]
0
20
40
60
80
100
120
140
160
180
0.5
1
1.5
2


π,ρ
N
N


$\tilde{V}_T(r)$[MeV]
r[fm]
50
0
−50
−100
−150
0.0
0.5
1.0
1.5
2.0
2.5
3.0
n=0
n=n_0
n=1.75n_0
n=3.3n_0
n=3.5n_0



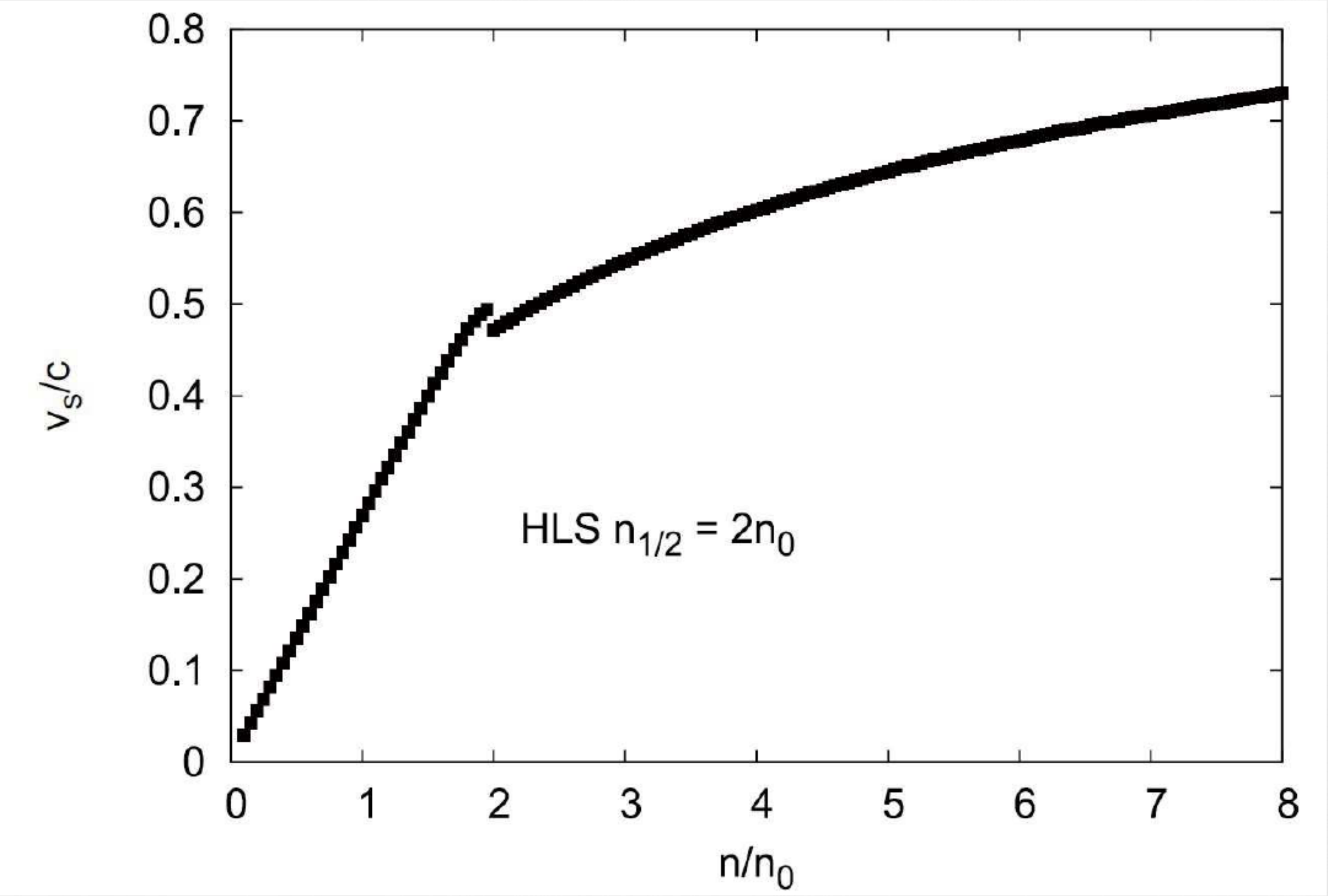




$$I \simeq (0.237 \pm 0.008) M R^2 [1 + 4.2 \frac{M}{M_\odot} \frac{km}{R} + 90 (\frac{M}{M_\odot} \frac{km}{R})^4]$$

J. M. Lattimer and B. F. Schutz, Astrophys. J. **629**, 979 (2005).

Sound velocity

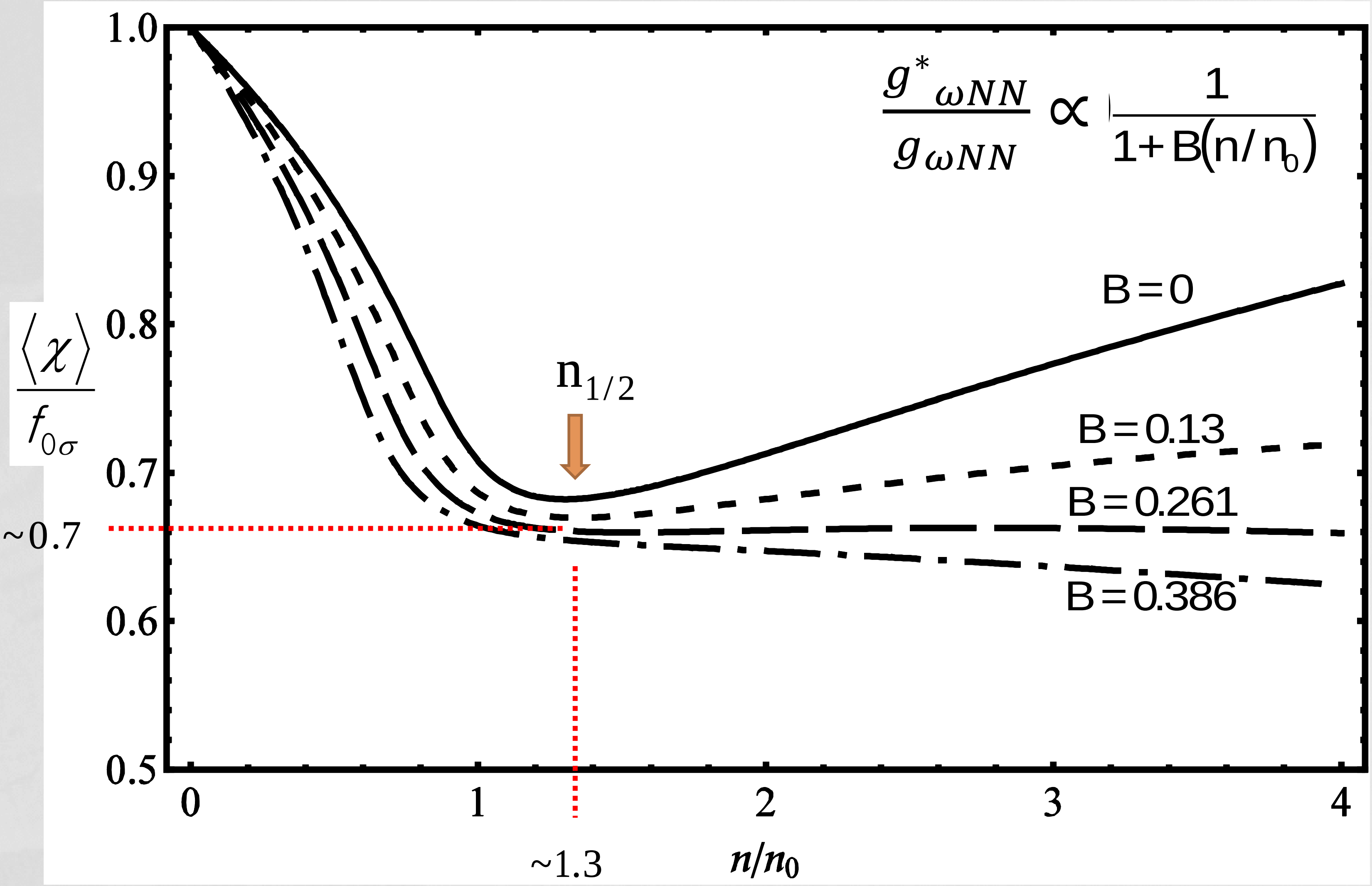


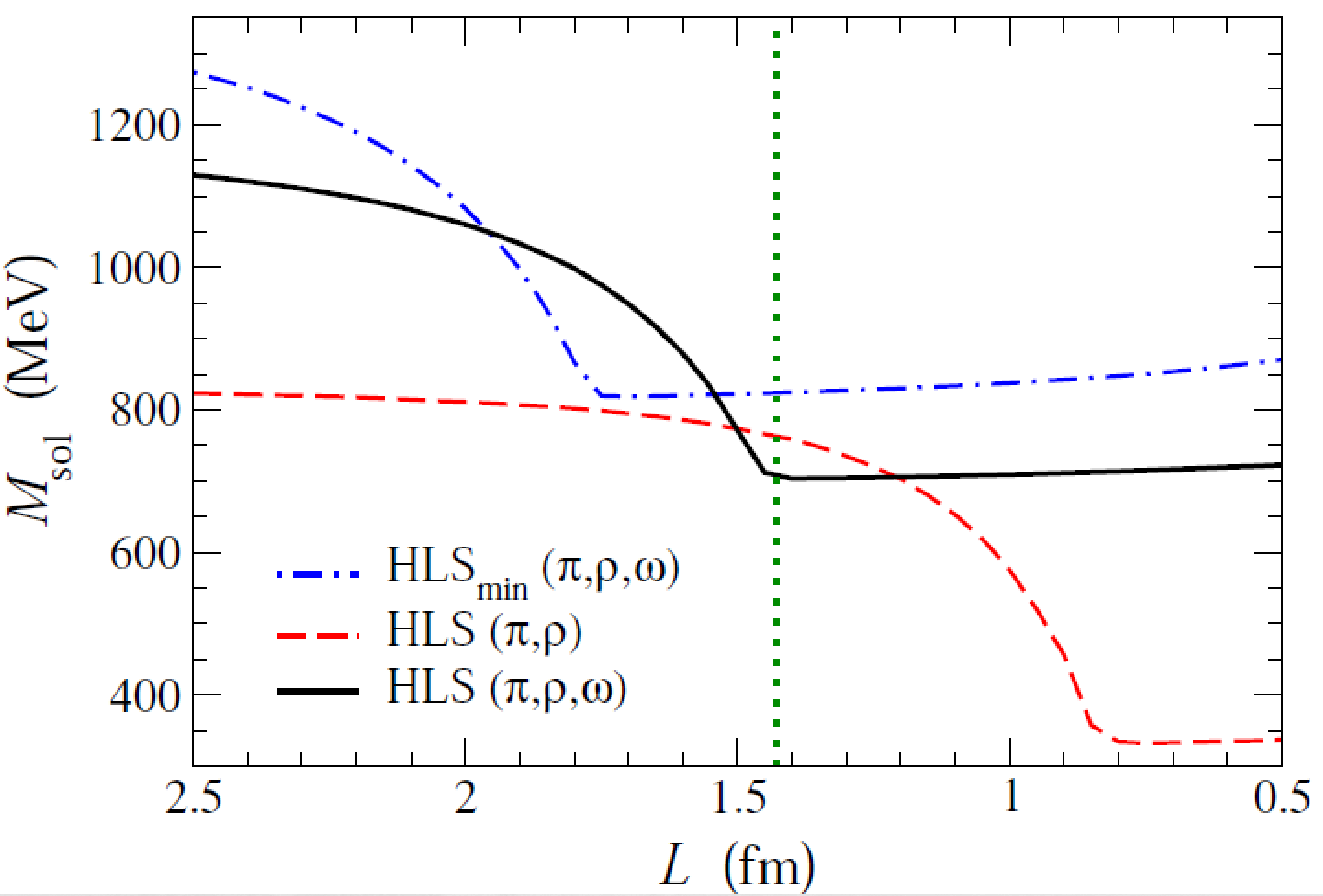
Summary

- We applied the scaling behavior of the parameters in the effective model to the nuclear matter by using V_{lowk}, which predicts the large amount of the nucleon and scalar meson masses stay more or less constant.
- What we found is that the scaling of the parameters predicts the soft EoS for the symmetric nuclear matter but the stiff EoS for the neutron matter. And, the symmetry energy is soft in low density region, but it is stiff in high density region.
- We argue that the change of the scaling behavior changes the EoS to be stiff from soft.

L. Tolos, I. Sagert, D. Chatterjee, J. Schaffner-Bielich and C. Sturm, "Implications for compact stars of a soft nuclear equation of state from heavy-ion data," PoS NICXII , 036 (2012) [arXiv:1211.0427 [astro-ph.HE]].

The calculation was done in the mean-field approximation.
[W.-G. Paeng, H. K. Lee, M. Rho and C. Sasaki, Phys. Rev. D 88, 105019 (2013)]





which is supported by the skyrmion calculation in a crystal
[Y.-L. Ma, M. Harada, H. K. Lee, Y. Oh, B.-Y. Park and M. Rho, Phys. Rev. D 88, 014016 (2013)]

$$
\begin{aligned}
\mathcal{L}_{bs\mathrm{HLS}}(\pi,\chi,V_\mu,N) &= \left(\frac{\chi}{f_{0\sigma}}\right)^2\left(f_{0\pi}^2 \mathrm{tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}_{\perp}^{\mu}\right] + f_{0\sigma_\rho}^2 \mathrm{tr}\left[\hat{\alpha}_{\|\mu}\hat{\alpha}_{\|}^{\mu}\right] + f_0^2 \mathrm{tr}\left[\hat{\alpha}_{\|\mu}\right]\mathrm{tr}\left[\hat{\alpha}_{\|}^{\mu}\right]\right) \\
&+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\chi) + \frac{f_{0\pi}^2}{4}\left(\frac{\chi}{f_{0\sigma}}\right)^3 \mathrm{Tr}(MU^\dagger + h.c.) + \bar{N}i\not{D}N - m_N\frac{\chi}{f_{0\sigma}}\bar{N}N \\
&+ g_A\bar{N}\gamma^\mu\gamma_5\hat{\alpha}_{\perp\mu}N + g_{\rho N}\bar{N}\gamma^\mu\hat{\alpha}_{\|\mu}N + g_{V0}\bar{N}\gamma^\mu \mathrm{tr}\left[\hat{\alpha}_{\|\mu}\right]N + \cdots
\end{aligned}
$$

with $D_\mu N = \left(\partial_\mu - ig_\rho\frac{\vec{\rho}_\mu\cdot\vec{\tau}}{2} - ig_\omega\frac{\omega_\mu}{2}\right)$, $g_{V0} = \frac{1}{2}\left(g_{\omega N} - g_{\rho N}\right)$ and $f_0^2 = \frac{1}{2}\left(f_{0\sigma_\omega}^2 - f_{0\sigma_\rho}^2\right)$, where M is the spurion field with $\langle M\rangle = 2B^0\begin{pmatrix} m_u & 0 \\ 0 & m_d\end{pmatrix}$.

Under HLS transformation,

$$
\begin{aligned}
\alpha_{\perp,\|}^\mu &\to u(x)h(x)\alpha_{\perp,\|}^\mu u(x)^\dagger h(x)^\dagger\,, \\
N &\to u(x)h(x)N \quad \& \quad \chi \to \chi
\end{aligned}
$$

and

$$
M \to g_L M g_R^\dagger \quad \& \quad U \to g_L U g_R^\dagger\,,
$$

where $g_{L,R} \in [SU(2)_L \times SU(2)_R]_{global}$, $h(x) \in [SU(2)_V]_{local}$ and $u(x) \in [U(1)_V]_{local}$. Under scale transformation,

$$
\begin{aligned}
\partial,\, N,\, \alpha_{\perp,\|},\, M,\, \chi &\to \lambda\left(\partial,\, N,\, \alpha_{\perp,\|},\, M,\, \chi\right)\,, \\
U &\to U\,.
\end{aligned}
$$